

LASER GYROSCOPES

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Correlation relationship between the dissipative and conservative backscattering components in the ring resonator of a laser gyroscope

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Abstract. **We report the results of measuring the complex coupling coefficients in ring He – Ne lasers with a radiation wavelength of 632.8 nm. It is found that in assembling ring resonators from mirrors of the same quality, there is no correlation between the dissipative and conservative backscattering components. The analysis of the measurement results shows that this behaviour is associated with the speckle structure of the backscattered field in the ring resonator. The presence of backscattering (BS) sources with a random phase on the mirrors' surface leads to a spread in the values of the BS parameters, which is described by the Rayleigh distribution function. Ways to reduce the spread in the values of dissipative and conservative backscattering components in assembling the ring resonators are discussed.**

Keywords: *ring laser, ring resonator, laser gyroscope, backscattering, lock-in threshold, complex coupling coefficients of counterpropagating waves, dissipative and conservative components of backscattering.*

1. Introduction

Backscattering (BS) on ring resonator (RR) mirrors is one of the main sources of errors in a laser gyroscope (LG) based on a ring He – Ne laser (RL) with a radiation wavelength of 632.8 nm. This scattering leads to the mutual coupling of counterpropagating laser waves and is the reason for the existence of a dead zone in measuring slow angular velocities of rotation (the so-called lock-in zone), and also causes nonlinear distortions of the LG scale factor [1 – 4]. It is no coincidence that LG developers spend significant resources on improving the methods for polishing mirror substrates and technologies for applying multilayer dielectric coatings.

In ring gas laser theory [1 – 4], the BS effect on the amplitudes and phase difference of the counterpropagating waves is described using two linear complex coupling coefficients (CCCs), which characterise part of the field falling into the counterpropagating wave as a result of the process of light scattering on the inhomogeneities of mirrors:

$$\boldsymbol{r}_{\mathrm{cw,\,ccw}} = r_{\mathrm{cw,\,ccw}} \exp(\mathrm{i}\varphi_{\mathrm{cw,\,ccw}}), \qquad (1)$$

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where $r_{\mathrm{cw,\,ccw}}$ are the CCC moduli for waves propagating in the clockwise and counter-clockwise directions; and $\varphi_{\mathrm{cw,\,ccw}}$ are the corresponding phase shifts resulting from backscattering.

From the structure of the equations describing the amplitudes and phase differences of counterpropagating waves, it follows that the phase shifts due to backscattering are present in them as a sum of the phase shifts: $\varphi = \varphi_{\mathrm{cw}} + \varphi_{\mathrm{ccw}}$. In this regard, the CCCs are characterised not by four parameters (two moduli and two phase shifts), but by three (two moduli and a total phase shift due to backscattering).

In the analysis of the system of equations for the amplitudes and phases of counterpropagating RL waves, two types of BS sources are particularly distinguished, i.e. dissipative and conservative [3], which differ in the phase shift magnitude. Dissipative BS sources have a phase shift $\varphi = 2\pi$, which in this case is equivalent to $\varphi = 0$. The phase shift of conservative BS sources is π. The physical meaning of this difference in phase shifts is related to the fact that the conservative BS is due to inhomogeneities of the refractive index of the mirror material, while the dissipative BS is due to inhomogeneities in its absorption index [1].

Dissipative and conservative BS sources have different effects on nonlinear distortions of the LG amplitude-frequency response. For example, in the intensities of the counterpropagating RL waves, the modulation components with the beat frequency of the counterpropagating waves are observed. In the case when the RL frequency bias Ω significantly exceeds the lock-in threshold, the intensities of the radiation emerging from the laser can be represented by the relations:

$$I_{\mathrm{cw}} = I_{\mathrm{cw}}^{(0)}[1 + m_{\mathrm{cw}} \sin(\Omega t)], \qquad (2)$$

$$I_{\mathrm{ccw}} = I_{\mathrm{ccw}}^{(0)}[1 + m_{\mathrm{ccw}} \sin(\Omega t + \chi)], \qquad (3)$$

where $m_{\mathrm{cw,\,ccw}}$ are the modulation intensity depths usually constituting a fraction of a percent; $I_{\mathrm{cw,ccw}}^{(0)}$ are the constant intensity components of counterpropagating waves; and χ is the phase difference of the modulation components. In the case of conservative BS, antiphase modulation components are observed, and $\chi = \pi$. Dissipative BS results in-phase intensity modulations ($\chi = 0$). It should also be noted that the modulation depths due to the dissipative and conservative BS do not differ for waves of opposite directions.

In reality, the BS wave fields are formed by both types of sources, which have a rather complex effect on the distortions of the LG amplitude – frequency response. In particular, the CCC moduli can differ significantly for counterpropagating waves, the total phase shift caused by BS varies over a wide range of values [5].

In the case of a constant-bias LG, it is possible to obtain an analytical dependence of the nonlinear correction of the LG scale factor ΔK on the LG rotation velocity Ω, which makes allowance for the effect of both types of BS sources and is in good agreement with the measurement results [1, 2, 4]:

$$\Delta K = K - 1 = -\frac{S_+^2}{2\Omega^2} + \frac{S_-^2}{2(\Omega_g^2 + \Omega^2)}, \quad (4)$$

where Ω_g is the RL limit cycle strength; and S_+ and S_- describe the dissipative and conservative components of backscattering, respectively:

$$S_{+,-} = \frac{c}{L}\sqrt{r_{cw}^2 + r_{ccw}^2 \pm 2r_{cw}r_{ccw}\cos\varphi}. \quad (5)$$

The plus sign in the right-hand side of (5) corresponds to the dissipative BS component. Relation (4) is valid for the weak coupling regime of counterpropagating waves, when the constant bias Ω significantly exceeds the parameter S_+, which is the lock-in threshold (LT) Ω_L. The moduli of the dissipative (r) and conservative (R) CCCs components are related to the parameters $S_{+,-}$ as follows:

$$S_+ = \Omega_L = 2\frac{c}{L}r, \quad (6)$$

$$S_- = 2\frac{c}{L}R, \quad (7)$$

where c is the speed of light; and L is the RR perimeter. In this case, the parameters $S_{+,-}$ have the dimension of rad s^{-1}. Depending on the conditions of the problem (for example, in direct CCC measurements in RR [5]), the conservative and dissipative components can be understood both as the parameters $S_{+,-}$ and the moduli r and R. Accordingly, their dimensions can be Hz, rad s^{-1}, deg s^{-1}, or ppm.

Relation (4) is used to determine the values of these parameters when measuring nonlinear corrections to the LG scale factor. In the case of using multilayer dielectric mirrors with $Ta_2O_5-SiO_2$ and TiO_2-SiO_2 layers, the conservative component is three to seven times higher than the dissipative one. As a result, the scale factor correction at a large bias value Ω, as a rule, has a positive sign, and the phase difference of the intensity modulation components is close to π.

Note that when presenting the results of CCC measurements in the LG, the phrase ‘as a rule’ has to be used quite often. This is due to the fact that the laser radiation field scattered by the mirror is a complex structure of bright and dark bands and spots (a so-called speckle structure [6]). The total modulus of the dissipative component is represented as a scalar sum of individual dissipative BS sources:

$$r = |\sum r_n \exp(2ikl_n)|, \quad (8)$$

where r_n is the partial modulus of the coupling coefficient of the BS source; l_n is the longitudinal coordinate of the source along the RR optical axis; and k is the wave number. The expression for the total value of the conservative BS component can be presented in a similar way, in the form of a scalar sum of the coupling coefficients of individual conservative sources.

The result of the interference of BS sources with a random phase is a large spread in the values of the total CCC modulus. As an example, we present a histogram of lock-in thresholds for about 250 LGs assembled using mirrors with $Ta_2O_5-SiO_2$ layers (integral scattering coefficient of 10–15 ppm) [7]. The perimeter of the four-mirror resonator was 16 cm.

The LT histogram (Fig. 1) can be described with the Rayleigh distribution which is used to describe the statistics of the resulting field of coherent oscillators with a random phase value [8]. The probability density of this distribution is described by the expression

$$f(\Omega_L) = \frac{\Omega_L}{\sigma^2}\exp\left(-\frac{\Omega_L^2}{2\sigma^2}\right), \quad (9)$$

where σ is the scale parameter, approximately equal to the mean value of the lock-in threshold in the LG. This distribution is shown in Fig. 1 as a solid line. The presence of a negative quadratic exponent index in (9) does not allow us to explain the appearance in the histogram in Fig. 1 of an LG with a lock-in threshold exceeding the mean value by more than three times. This is due to the presence of large dust-like particles in the working area of the mirrors, which are responsible for large values of the lock-in threshold. In this case, the threshold values for about 15% of the assembled gyroscopes deviated from the Rayleigh distribution.

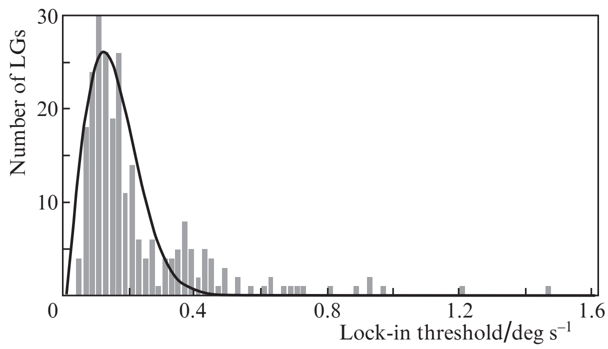

Figure 1. Histogram of the distribution of 250 LGs by the LT value in the range of 0.05–1.5 deg s^{-1}, consisting of 70 intervals of the LT values.

A similar behaviour can be expected from the histograms of the distribution of the conservative BS component. In this case, a natural question arises as to the correlation relationship between the BS components. The developers of modern LGs *a priori* believe that such a relationship exists, i.e., a decrease in the BS intensity guarantees a decrease in the lock-in threshold. It is no coincidence that modern methods of quality control of LG mirrors are also based on the fulfilment of this requirement. These include visual inspection of the surfaces of mirror substrates for the presence of point defects and shiny spots, measurement of surface roughness using a phase interferometer or an atomic force microscope, as well as measurements of the total integral scattering (TIS) of mirrors.

The authors of the patent [9], who proposed to significantly reduce the lock-in threshold at a constant TIS value of the mirrors, also relied upon this *a priori* statement. For this purpose, a natural oscillation is excited in the tuned RR using external laser radiation. In the opposite direction, the intensity of the counterpropagating wave caused by the backscattering of the tuned mirror is measured. By moving the mirror along the contact surface of the monoblock housing (mainly by turning it), a noticeable decrease in the BS intensity can be achieved, bringing the dark speckle spot closer to the direction of the counter mode propagation. The characteristic angular scale of the speckle field variation is $\lambda/2w$ (w is the RR waist radius), i.e., several arc minutes.

In this work, we essentially aimed at revealing the presence or absence of a correlation relationship between the dissipative and conservative BS components. For this, we conducted simultaneous measurements of both BS components when assembling the LG from mirrors of approximately the same quality. The method is based on the measurement of nonlinear distortions of the LG scale factor with a rectangular bias and is described in detail in our previous work [10].

2. Measurement technique and results

Measurements of the dissipative and conservative BS components were performed using standard measuring equipment developed at the 'Polyus' Research Institute. The measurement technique is based on the analysis of the dependence of the nonlinear distortions of the scale factor ΔK on the angular rotation velocity Ω of a Zeeman LG with a rectangular bias. The technique is described in detail in [10]. When determining S_+ and S_-, the following relation was used:

$$\Delta K = \frac{S_+^2}{2(\Omega_0^2 - \Omega^2)} + \frac{S_-^2(\Omega_g^2 + \Omega^2 - \Omega_0^2)}{2[(\Omega - \Omega_0)^2 + \Omega_g^2][(\Omega + \Omega_0)^2 + \Omega_g^2]}, \quad (10)$$

where Ω_0 is the rectangular bias amplitude. This relation is valid for describing the wings of the dependence $\Delta K(\Omega)$ at $|\Omega - \Omega_0| > (1-2)$ deg s^{-1}.

The measurements were carried out with single-axis ZLK-20 [11] sensors. The sensors were installed on a turntable, the relative deviation of the rotation velocity of which did not exceed 0.001%. The measurement error of the nonlinear correction ΔK was no more than 2–3 ppm. The range of changes in the table rotation velocity was 10–400 deg s^{-1}.

The ring resonators of the sensors were assembled from two types of mirrors with $Ta_2O_5-SiO_2$ and TiO_2-SiO_2 layers. The TIS of the mirrors selected for the assembly ranged from 10 to 15 ppm. As many as 205 sensors were tested. The values of the parameters S_+ and S_- were measured for four adjacent modes, the difference in the natural frequencies of which in the RR with a nonplanar contour was $c/2L$. Thus, a data array of 820 pairs of values of the parameters $S_{+,-}$ was formed, the histograms of which are shown in Fig. 2.

A significant spread of the measured values of the BS components was recorded. For conservative components, the ratio of the maximum and minimum values of the S_- parameter was ~20 (0.88 and 0.046 deg s^{-1}), while for S_+ it was ~7 (0.36 and 0.053 deg s^{-1}).

We tried to approximate the obtained histograms with the Rayleigh distribution using relation (9), where the mean values of S_+ and S_- were used as the scale parameter. In the case of a conservative component, this ensured a good qualitative coincidence (Fig. 2a).

For the dissipative component, there are striking differences from the Rayleigh distribution. First of all, this applies

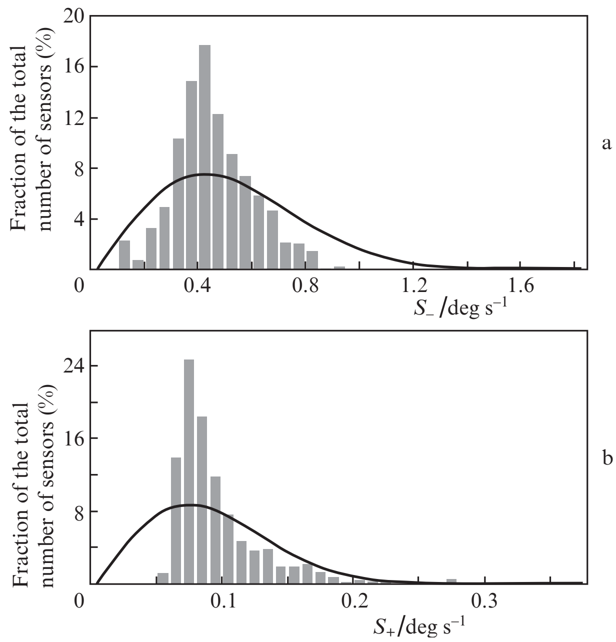

Figure 2. Histograms of 820 values of the parameters (a) S_- and (b) S_+ with a sampling interval of 0.015 deg s^{-1}. The ordinate axis shows the proportion of the total number of sensors falling within the given range of values. The solid line is the result of the approximation using the Rayleigh distribution.

to the region of small (lower than mean) values of S_+. According to relation (9), the probability density for small values of the parameter (in this case S_+) is directly proportional to its value. Numerical estimates of the proportion of sensors with S_+ value, which is three times less than the mean value, show that there should be at least 6%–7% of the total number of sensors. In fact, we have not recorded any such sensor. The reason for this difference is not yet clear to us. This may be due to the BS effect from a photodetector installed at the output mirror with a transmittance greater than 1000 ppm.

To answer the question about the correlation relationship between the measured values of S_+ and S_-, we use the Pearson criterion [12], in which the correlation coefficient of two random variables is estimated as follows:

$$r_{x,y} = \frac{\sum (x_n - \langle x \rangle)(y_n - \langle y \rangle)}{\sqrt{\sum (x_n - \langle x \rangle)^2 \sum (y_n - \langle y \rangle)^2}}, \quad (11)$$

where x_n and y_n are random variables (in this case, S_- and S_+); and $\langle x \rangle$ and $\langle y \rangle$ are their mean values.

The correlation coefficient of the parameters S_- and S_+ is 0.07 in our case, which indicates that there is no correlation relationship between the conservative and dissipative BS components. This significantly complicates the process of RR assembly and alignment. In particular, a decrease in the BS intensity from the mirror in the opposite direction is not always accompanied by a decrease in the lock-in threshold.

Another important conclusion that follows from the results of this experiment suggests that the CCC separation into two components is quite correct. The absence of a correlation relationship between them allows us to consider them as two independent BS parameters.

When analysing the S_- and S_+ statistics, their mean values are of importance. In our experiments, $\langle S_- \rangle = 0.4$ deg s^{-1} and

$\langle S_+ \rangle = 0.069$ deg s^{-1}. The conservative component value can be estimated based on the isotropic BS model, in which the BS wave intensity is determined by the TIS and the solid angle of the RR mode. This problem was solved in [13]. To estimate the CCC modulus and the RR solid angle, the following relations are used:

$$r = \sqrt{\frac{S\Phi}{4\pi}}, \tag{12}$$

$$\Phi = \left(\frac{w}{4L}\right)^2, \tag{13}$$

where w is the waist radius of the RR fundamental mode; S is the TIS value of the mirror; and Φ is the RR solid angle. Two important simplifications have been made in this case. Firstly, when calculating the conservative component, we neglect the contribution of the dissipative component to the BS wave intensity, which does not exceed 10%. Secondly, we assume that all BS sources are located on the same RR mirror, i.e., the TIS value appearing in relation (12) must be multiplied by four (in accordance to the number of RR mirrors). Given the sensor scale factor, for the RR parameters used in our experiments (L = 200 mm, w = 0.36 mm, S = 12 ppm) we obtain $\langle S \rangle = 0.34$ deg s^{-1}. To estimate the mean value of the dissipative component, we divide this value by the ratio $\langle S_- \rangle / \langle S_+ \rangle$ found above and obtain $\langle S_+ \rangle = 0.058$ deg s^{-1}, which is in good agreement with the results of our measurements.

In conclusion of this section, we present a histogram (Fig. 3) of the distribution of the nonlinear correction ΔK. Relation (10) describing the dependence $\Delta K(\Omega)$ consists of two terms, one of which is determined by the conservative BS component, and the other, by the dissipative one. In an LG with a rectangular bias, these terms are approximately equal over a wide range of rotation velocities. Thus, the dependence $\Delta K(\Omega)$ is determined by two random parameters: S_+ and S_-. Probably, this feature was the reason for the good coincidence of the histogram in Fig. 3 with its approximation by the Rayleigh distribution.

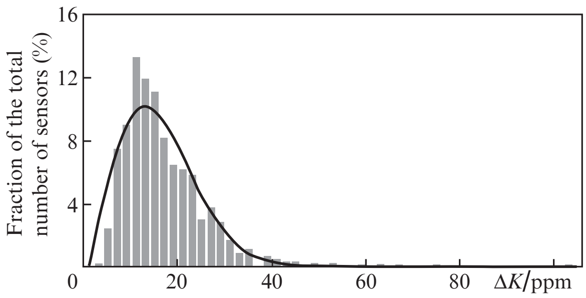

Figure 3. Histogram of the nonlinear correction to the LG scale factor with a sampling interval of 2 ppm at Ω = 18 deg s^{-1}.

3. Discussion of results and conclusions

The results of our experiments revealed the absence of a correlation relationship between the dissipative and conservative BS components in the LG ring resonator. In the language of statistics, this indicates the presence of two independent arrays of BS sources in the mirror workspace. If we switch to the language that is more commonly used by LG developers, we can say that high BS intensity in the RR does not necessarily lead to a large lock-in threshold in the LG. Conversely, at low BS intensity, the lock-in threshold need not be low.

Relying on a mirror quality assessment based only on measurements of the light intensity scattered by the mirror, it is rather difficult to arrive at such a paradoxical conclusion. This is due to the fact that the scattered light intensity is determined simultaneously by both BS sources. In this case, the contribution of dissipative sources to the balance of integral light scattering by the mirror is only a few percent. Consequently, if there is really a correlation relationship of the dissipative and conservative BS components with BS intensity, this can only be expected for the conservative component.

Direct CCC measurements [5] allow us to understand the physical essence of the backscattering phenomenon in the RR. As an illustration, we present the results of our experiments. Figure 4 shows the results of measuring the values of 24 pairs of CCC moduli (measured in one direction according to the method of work [5]) and the LG lock-in thresholds. The experiments were performed using four-mirror RLs with a flat contour and L = 28 cm. The RR assembly used mirrors with $Ta_2O_5-SiO_2$ layers having an TIS value of approximately 20 ppm. The measurement scheme of the patent [9] was reproduced with one difference: it was not the BS intensity that was measured, but the CCC modulus (it is proportional to the square root of the intensity).

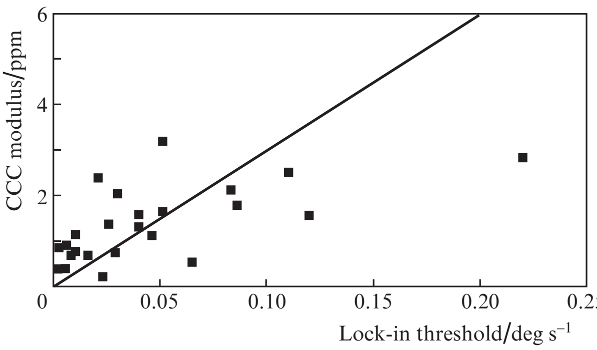

Figure 4. The results of measurements of the CCC moduli in the RR using the method from work [5] and the lock-in threshold in the RL with a perimeter of the four-mirror RR of 28 cm. The solid line corresponds to the linear relationship between these parameters in the case of their complete correlation.

It can be seen that there is no correlation between these parameters; therefore, the alignment method in [9], which, according to its authors, makes it possible to minimise the LG lock-in threshold, seems to us incorrect. However, the main idea of the authors on using the features of the speckle structure of BS fields in the RR alignment seems to us promising. It is only necessary to switch from BS intensity measurements to CCC measurements in the RR being aligned.

Now a few words about the speckle structure of BS fields. In our opinion, its presence is an obvious consequence of inhomogeneities in the structure of multilayer dielectric coatings. Figure 5a shows a fragment of a photograph of a mirror with a TIS of about 10 ppm, obtained using a dark-field microscope with a 500-fold magnification. The image fragment size was ~25 × 25 μm. After the shooting, additional digital image processing was performed. We have a rather complex picture of inhomogeneities randomly located on the mirror surface and in the depth of the layers of its multilayer coating. The random variables here are not only the coordinates of the location of inhomogeneities, but also the intensities of the scattered light. From the viewpoint of the formation of CCC moduli [see (8)], such a structure leads to histograms described by the Rayleigh distribution.

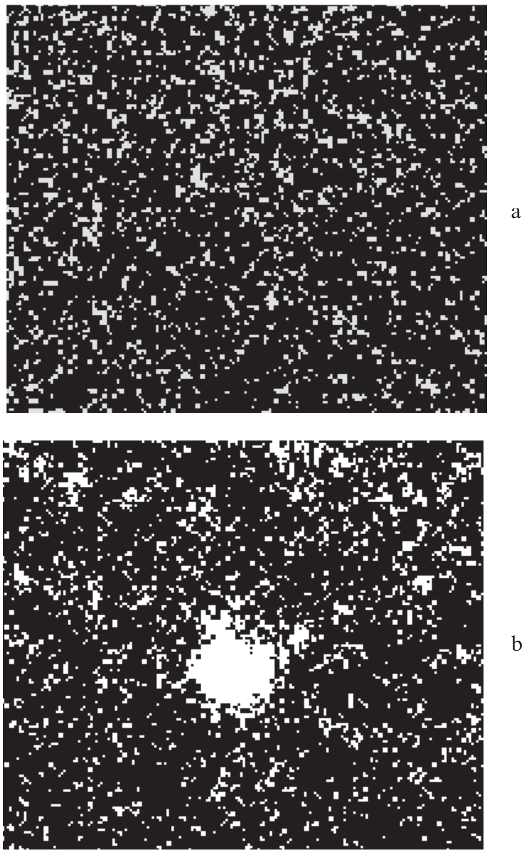

Figure 5. Fragments of photographs of mirrors, obtained using a dark-field microscope with a 500-fold magnification. Image size is ~25 × 25 μm; Figures a and b differ in the presence of large point defects in Fig. b.

The appearance on the mirror surface of large (micron-sized) point defects (Fig. 5b) can significantly distort the result of summation of partial CCCs. In the language of the Rayleigh distribution, an individual large inhomogeneity is an array of inhomogeneities with the same phase [see (8)], which noticeably increases the value of the total CCC modulus of the BS source array.

Let us illustrate this statement by the example of a simple model of an array of BS point sources We place 1000 BS point sources with the same partial CCC value on the working surface of the mirror. For definiteness, we assume that these are dissipative BS sources. A partial contribution to the lock-in threshold of each of the sources is 1 Hz. Using a random number generator, we assign each of the 1000 BS sources its own phase shift value $2kl_n$, which lies in the range from zero to 2π.

Figure 6 shows a histogram of the distribution of lock-in thresholds for an array of point sources. The number of realisations of random number combinations in this case was 10^6. The envelope of this histogram has the form of a Rayleigh distribution. It is noteworthy that the mean value of the lock-in threshold of the array of sources was ~20 Hz, whereas with the same phase shift, the lock-in threshold would be equal to the sum of the partial values (1000 Hz).

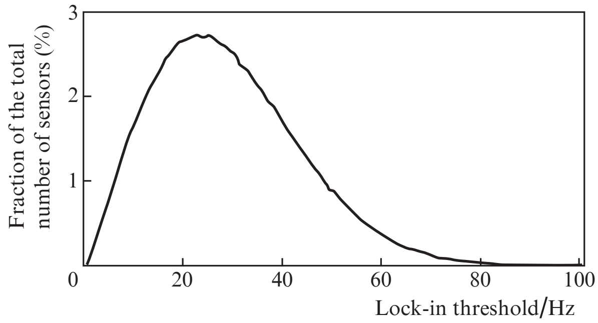

Figure 6. Envelope of the distribution histogram (model) of the lock-in threshold of 1000 dissipative BS point sources with a random phase. The partial value of the lock-in threshold is 1 Hz, the number of realisations is 10^6.

In other words, if we imagine that individual inhomogeneities of the mirror merge into one large defect, the result of this merging will be an increase in the CCC modulus by almost two orders of magnitude. Therefore, the appearance of large inhomogeneities on the surface of the LG mirrors significantly affects the shape of the histograms of the conservative and dissipative BS components. In particular, the fact that sensors with small (compared to the mean value) values of these parameters are absent in the histograms (see Figs 2 and 3) can be explained by the presence of large inhomogeneities (for example, dust particles) on the mirrors' surface.

By the way, point inhomogeneities in mirror coatings can be considered as possible candidates for dissipative BS sources. We have already mentioned in the Introduction that large dust-like particles fall into the working area of the LG mirrors, which lead to high values of the LG lock-in thresholds. In mirrors with ion-plasma sputtering, the density of submicron-sized defects can amount to several tens of defects per mm^2. The contribution of these defects is not negligible and, as a rule, exceeds 10% of the TIS value. In this regard, point defects should be taken into account when assessing the balance of CCC values. As for their dissipative nature, this is just our assumption that goes beyond the scope of the issues discussed here.

Many questions also arise when optimising technological processes for the production of LG mirrors, since the BS parameters are technological derivatives. For example, it is known that the TIS value of a mirror is related to the surface roughness of its substrate. And to what extent is the ratio of the mean values of the dissipative and conservative BS components (1/5) a universal function of the technological process? No less interesting is the answer to the question regarding the effect of UV radiation from a gas discharge on the BS sources. Is it possible to 'transform' conservative sources into dissipative ones under its influence? Do the CCCs of the RR remain unchanged after performing electrovacuum treatment of LG sensors?

Of course, within the framework of this work, we can only ask these questions. Answers to them can be obtained from the results of direct CCC measurements in the RR of a laser gyroscope [5]. This will make it possible to control the values of these parameters at all stages of LG production, from assembling and aligning of the RR to vacuum processing and testing of gyroscopic sensors.

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