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Generation of superstrong quasi-stationary magnetic fields in laser cluster plasma

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Abstract. **An analytical model has been developed for the generation of a superstrong quasi-stationary magnetic field (up to several GG) in the focal waist of an ultra-intense short circularly polarised laser pulse interacting with a gas-cluster target. The rotation of relativistic electrons around the ionised core of the cluster has been shown to produce a magnetic moment and a quasi-stationary magnetic field. With a large number of nanoclusters, the magnetic field occupies the entire focal volume with a characteristic spatial scale of tens of micrometers and exists during the cluster expansion time (i.e., several picoseconds).**

Keywords: *superstrong quasi-stationary magnetic field, nanoclusters, superintense laser interaction.*

1. Introduction

Studies of the influence of superstrong magnetic fields on the properties of objects are relevant in various fields of physics and astrophysics. The experimental generation of such fields is possible at high current densities, achieved, for example, using Z-pinches [1]. Recently, the pinch effect in laser targets made of thin wires has been investigated and a magnetic field amplitude of the order of 1 GG has been achieved on a scale of a few micrometers [2, 3]. Other methods of generating magnetic fields of the same or higher intensity are also of considerable interest. It is known that the absorption of a relativistic-intensity laser pulse by targets of various structures is accompanied by the generation of a current of hot electrons and a countercurrent of colder electrons generating quasi-stationary magnetic fields of large amplitude [4–7]. In the case of solid targets, the magnetic field is localised near the target surface [8] and its strength is tenths of the laser field strength, while the lifetime is much longer than the laser pulse duration. In targets transparent to laser radiation, a circularly polarised laser pulse due to the inverse Faraday effect [9, 10] generates longitudinal (along the laser beam axis) magnetic fields, the lines of force of which correspond to the magnetic dipole occupying the focal volume. A special laser pulse with a screw-shaped spatial intensity distribution was proposed to obtain a longitudinal magnetic field with an amplitude above 1 GG [11]. In addition to the inverse Faraday effect, a quasi-stationary magnetic field is generated in a gas laser target behind the leading edge of a short laser pulse due to the anisotropy of the electron pressure during tunnelling ionisation of atoms [12]. However, these methods can produce a short-lived (on the order of the laser pulse duration) magnetic field in a rarefied plasma. When use is made of long (several picosecond) laser pulses of relativistic intensity and spiral metal targets, it is possible to generate a quasi-stationary field with an amplitude of $\sim$10 MG on the spiral axis [13]. The question arises whether it is possible, under laboratory conditions, to further increase the amplitude of the magnetic field, its lifetime, and the volume of space occupied by the field.

In this work, we have developed a theory of generation of superstrong magnetic fields and giant magnetic moments, based on electron inertia in cluster-gas targets [14, 15] irradiated by a circularly polarised ultrashort laser pulse of relativistic intensity. In contrast to a homogeneous low-density plasma, our method makes it possible to generate many dense plasmas (magnetic dipoles) equal to the number of clusters in the focal volume. For an optimal cluster size, a large number of clusters in the focal region produce a quasi-uniform magnetic field throughout the focal volume. Thus, a medium is formed from parallel oriented magnetic dipoles, in which the magnetic field occupies the entire volume between the clusters and leads to their magnetic interaction. In this case, the field lines of force can close through the external space (when the entire focal region is equivalent to one giant dipole). A unique feature of an individual element of such a structure (a magnetic dipole on a scale of hundreds of nanometres) is a toroidal long-lived relativistic electric current that generated a dipole, which is one of the areas of research in modern electrodynamics [16].

We simulated the interaction of a single cluster with a circularly polarised laser pulse using numerical methods in [17]. In a subsequent work [18], we analytically estimated the values of the magnetic moments of the clusters and the structure of the magnetic field of the cluster target, as well as proved the possibility of collective magnetic interaction of nanoclusters. This paper is devoted to the further development of the theory of magnetic field generation in a cluster target: the dependence of the magnetic field on the cluster radius and the duration of the laser pulse is taken into account, and the temporal dynamics of the appearance of a magnetic field during a laser pulse and its relaxation are investigated. The correctness of the presented theory is proved by comparison with

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numerical calculations [17]. Note that for the generation of magnetic dipoles, a laser pulse energy of the order of several or tens of millijoules is sufficient; therefore, a 'quasi-stationary' magnetisation of the focal volume of a cluster target is possible by repeating laser pulses with a frequency of up to several kHz. The thus generated magnetoactive cluster laser plasma with a superstrong regular magnetic field and magnetic dipoles can be used as an example of the experimental implementation of magnetoplasma structures of pulsars in astrophysics.

2. Dynamics of cluster electrons and transfer of the angular momentum from the laser field to cluster electrons

We assume that as a result of interaction with a circularly polarised electromagnetic (EM) wave, the cluster is partially ionised (with a charge Q) and electrons in the form of a spherical layer (with a total charge $-Q = eN_e$, where e is the electron charge and N_e is the number of electrons in a layer), surrounding the ionised core of a cluster of radius R, rotate under the action of a circularly polarised laser pulse and the cluster's internal electric and magnetic fields (see Fig. 1). In the electron shell of the cluster there is a radius p, where the electron density is equal to the critical density [$n_e(p) = n_{cr}$]; this radius is called the characteristic electron radius of the cluster. Then the thickness of the spherical layer of moving electrons, nontransparent for laser radiation, will be $p - R$. The thickness of the skin layer l_s in the ionised core of the cluster will be assumed to be small in comparison with R.

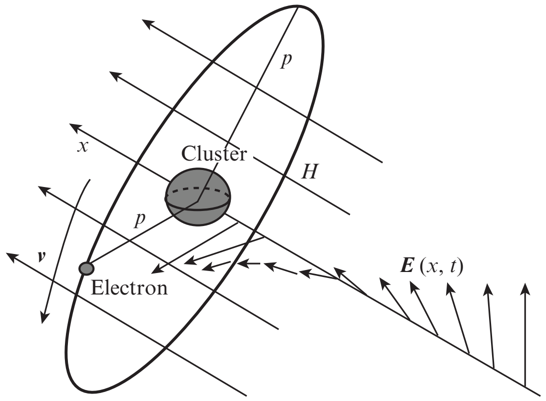

Figure 1. Scheme of interaction of a circularly polarised laser wave with a nanocluster.

Let us consider the dynamics of an electron in a quasi-stationary electric and magnetic field of a cluster and in the field of a circularly polarised wave $\boldsymbol{E}(x,t) = E_L\cos(kx-\omega t)\boldsymbol{e}_y + E_L\sin(kx-\omega t)\boldsymbol{e}_z$. The vector potential of such a wave has the form

$$\boldsymbol{A}(x,t) = \frac{E_0}{k}[\sin(kx-\omega t)\boldsymbol{e}_y - \cos(kx-\omega t)\boldsymbol{e}_z]. \quad (1)$$

The Lagrange function of a shell electron in the EM fields of a cluster and a laser pulse in a cylindrical coordinate system with the x axis along the axis of a circularly polarised laser beam is written as

$$L = -mc^2/\gamma + \frac{e}{c}\left[A_\alpha(r,x) - \frac{E_L}{k}\cos(kx-\omega t-\alpha)\right]r\dot{\alpha}$$
$$+\frac{eE_L}{\omega}\dot{r}\sin(kx-\omega t-\alpha) - e\varphi(r,x), \quad (2)$$

where $\gamma = 1/\sqrt{1-\beta^2}$, $(\beta c)^2 = \dot{x}^2 + \dot{r}^2 + (r\dot{\alpha})^2$, $e < 0$. The scalar potential

$$\varphi(r,x) = \frac{Q}{(r^2+x^2)^{1/2}} - \frac{Q(r^2+x^2+p^2)}{2p^3},$$
$$R \leqslant (r^2+x^2)^{1/2} \leqslant p,$$

corresponds to the electric field of the ionic core and the electron shell and vanishes (screening) at $r^2 + x^2 = p^2$. The angular component of the vector potential of a quasi-stationary magnetic field for a uniform spherical layer rotating with an angular velocity $\dot{\alpha}$ is determined by the expression

$$A_\alpha(r,x) = -\frac{Q\dot{\alpha}r}{2cp}\left[1 - \frac{3(r^2+x^2)}{5p^2}\right],$$
$$R \leqslant (r^2+x^2)^{1/2} \leqslant p. \quad (3)$$

Note that in formula (2) only the terms including the laser field depend on the angle of rotation α; therefore, the torque of forces $(\partial L/\partial\alpha)$ is produced by the laser field rather than by quasi-stationary fields. The equation of motion of an electron along the angle α (the equation for the angular momentum of an electron) takes the form

$$\frac{d}{dt}\left\{\gamma m r^2\dot{\alpha} - \frac{eQ\dot{\alpha}r^2}{2c^2p}\left[1 - \frac{3(r^2+x^2)}{5p^2}\right]\right\}$$
$$= eE_L r(t)[1 - \dot{x}(t)/c]\sin[kx(t) - \omega t - \alpha(t)]. \quad (4)$$

The second term under the sign of the derivative on the left-hand side of (4) represents the contribution to the total angular momentum of the system of a quasi-stationary magnetic field. Note that the ratio $(|e|Q/p)/(2\gamma mc^2)$ of the two terms under the sign of the derivative on the left-hand side coincides (up to a factor of the order of unity) with the ratio of the characteristic potential energy of interaction of an electron $|e|Q/p$ to its kinetic energy $2\gamma m_e c^2$. The condition for the boundedness of the orbit of the relativistic electron in the Coulomb field of the cluster after the end of the laser pulse is the fulfilment of the inequality $m_e c^2\gamma - |e|Q/p < m_e c^2$, i.e., for $\gamma > 1$, the ratio $(|e|Q/p)/(2\gamma mc^2) \sim 1/2$, and the contributions of the electron and the quasi-stationary field to the total angular momentum in (4) are approximately the same.

Radial equation of motion of an electron

$$\frac{d}{dt}\gamma m\dot{r} = \gamma m r\dot{\alpha}^2 + \frac{eQr}{(r^2+x^3)^{3/2}}\left[1 - \frac{\dot{\alpha}^2(r^2+x^3)^{3/2}}{c^2p}\right] +$$

$$+eE_{\rm L}\Big(1-\frac{\dot{x}}{c}\Big)\cos[kx(t)-\omega t-\alpha(t)] \quad (5)$$

shows that the radial motion occurs under the action of the centrifugal force $\gamma mr\dot{\alpha}^2$, the force of the Coulomb interaction with the ionic core, reduced by the value of the Lorentz force of the quasi-stationary magnetic field [these two forces correspond to the second term on the right-hand side of Eqn (5)], and the force from the side of the laser field [last term in (5)]. If (5) is averaged in time over several laser field periods [which corresponds to the search for a solution in the form $r(t)=\langle r\rangle+\delta r(t),\ \dot{\alpha}(t)=\langle\dot{\alpha}\rangle+\delta\alpha(t), x(t)=\langle x\rangle+\delta x(t)$)], then the average values of the total derivative on the left-hand side and the laser field force vanish, and the time-averaged radial motion corresponds to the compensation of the centrifugal and Coulomb forces acting on the electron:

$$\gamma m\langle r\rangle\langle\dot{\alpha}\rangle^2+\frac{eQ\langle r\rangle}{(\langle r\rangle^2+\langle x\rangle^3)^{3/2}}\times\left[1-\frac{\langle\dot{\alpha}\rangle^2(\langle r\rangle^2+\langle x\rangle^3)^{3/2}}{c^2p}\right]=0. \quad (6)$$

Since the electron rotation is caused by the action of a circularly polarised wave, we have $\langle\dot{\alpha}\rangle\approx-\omega$ [the direction of rotation of the wave electric field vector $\boldsymbol{E}=E_0\cos(kx-\omega t)\boldsymbol{e}_y+E_0\sin(kx-\omega t)\boldsymbol{e}_z$ corresponds to rotation from the z axis to the y axis, which is opposite to the direction of increasing the angle α, with the angle being measured from y to z]. The quantity $\langle r\rangle$, up to a numerical factor of the order of unity, coincides with the previously introduced electron radius of the cluster: $\langle r\rangle\approx p$.

The equation of motion of an electron in the longitudinal (x) direction, like equations (4) and (5), follows from the Lagrange function (2):

$$\frac{\rm d}{{\rm d}t}\gamma m\dot{x}=-\frac{\partial}{\partial x}\Big[-e\frac{r\dot{\alpha}}{c}A_\alpha(r,x)+e\varphi(r,x)+\frac{eE_{\rm L}}{\omega}\cos(kx-\omega t-\alpha)r\dot{\alpha}-\frac{eE_{\rm L}}{\omega}\dot{r}\sin(kx-\omega t-\alpha)\Big]. \quad (7)$$

By performing the averaging procedure [$x(t)=\langle x\rangle+\delta x(t)$], we obtain an equation for $\langle x\rangle$, corresponding to the equality of the force of the ponderomotive pressure of the laser wave and the force of the quasi-stationary field, the forces acting on the electron in the longitudinal direction:

$$\frac{eQ\langle x\rangle}{(p^2+\langle x\rangle^2)^{3/2}}\left[1+\frac{3p^2\omega^2}{5c^2}\right]+\frac{eE_{\rm L}p\omega}{c}\sin(k\langle x\rangle)=0. \quad (8)$$

In the weak-field approximation [$|e|E_{\rm L}/(m_{\rm e}\omega c)\ll 1$], we have $\langle x\rangle\approx 0$, which is equivalent to the minimum of the electrostatic potential energy at the centre of the ionic core. Note that equation (8) corresponds to the condition of minimum effective longitudinal potential energy of an electron, $U_{\rm eff}(x)=-epkA_\alpha(p,x)+e\varphi(p,x)+eE_{\rm L}p\cos(kx)\approx epk^2Q/5+eE_{\rm L}p-ek^2x^2Q(0.3+k^{-2}p^{-2}+E_{\rm L}p^2/2Q)/p$, which coincides with the energy of a harmonic oscillator ($e<0$), that is, the oscillating addition $\delta x(t)$ satisfies the equation of oscillations of a relativistic harmonic oscillator:

$$\frac{\rm d}{{\rm d}t}\gamma m\delta\dot{x}=-\frac{\partial U_{\rm eff}(\langle x\rangle+\delta x)}{\partial\langle x\rangle}\approx-\delta x\frac{\partial^2U_{\rm eff}(\langle x\rangle)}{\partial\langle x\rangle^2}=-\delta x\frac{|e|Qk^2}{p}\left(\frac{3}{10}+\frac{1}{kp}+\frac{E_{\rm L}p^2}{2Q}\right). \quad (9)$$

Thus, the motion of a cluster electron is the rotation of an electron in the transverse plane yz and a simultaneous vibration along the x axis relative to the value $\langle x\rangle$ with a frequency

$$\omega_1=\sqrt{\frac{|e|Qk^2}{pm\gamma}\left(\frac{3}{10}+\frac{1}{kp}+\frac{E_{\rm L}p^2}{2Q}\right)}.$$

This character of motion is close to the results of numerical simulation given in [17].

Equation (4), when performing the averaging procedure, turns into identity, since the mean value of the derivative on the left is equal to zero and the mean value of the periodic function on the right is also equal to zero. After the end of the laser pulse ($E_{\rm L}=0$), Eqn (4) expresses the law of conservation of the angular momentum of the electron: $\gamma mr^2\dot{\alpha}\approx\gamma mp^2\omega=$ const(t). To estimate the value of $\gamma mp^2\omega$, we use the laws of conservation of energy and angular momentum. Let us find the laser pulse energy absorbed by the cluster electrons:

$$E_{\rm abs}=|e|N_{\rm e}\int_0^{\tau_{\rm L}}\boldsymbol{E}_{\rm L}(t,x(t))\boldsymbol{v}_{\rm e}(t){\rm d}t+|e|N_{\rm e}\varphi(r(\tau_{\rm L}),x(\tau_{\rm L}))$$
$$=|e|N_{\rm e}E_{\rm L}\int_0^{\tau_{\rm L}}[\dot{r}(t)\cos(kx(t)-\omega t-\alpha(t))+r(t)\dot{\alpha}(t)\sin(kx(t)-\omega t-\alpha(t))]{\rm d}t+|e|N_{\rm e}\varphi(r(\tau_{\rm L}),x(\tau_{\rm L})). \quad (10)$$

The last term on the right-hand side of (10) describes the contribution of the energy of the electrostatic field of the cluster to the absorbed energy. Note that the potential φ depends on the total cluster charge Q, which is also determined by the value of $E_{\rm abs}$. Thus, expression (10) is an implicit equation with respect to $E_{\rm abs}$ or with respect to the cluster's absorption coefficient η, since $E_{\rm abs}$ is expressed in terms of the absorption coefficient and laser intensity I: $E_{\rm abs}=\eta I\pi p^2\tau_{\rm L}$. (The absorption coefficient of a spherical cluster is discussed in more detail in the Appendix and papers [19, 20].)

Note that the radius p of the critical electron density differs from the initial cluster radius R by several times. Indeed, let all electrons of the cluster be 'heated' by a laser pulse (which is an upper estimate, since the skin layer thickness $l_{\rm s}$ is less than R). Then, at an initial electron density of a cluster of $\sim 100\,n_{\rm cr}$, the value of p is estimated as $p\sim R\sqrt[3]{100}\approx 4.6\,R$. The absorbed laser energy in the case of circular polarisation of a laser pulse is related to the absorbed angular momentum by the expression $J_{\rm abs}=E_{\rm abs}/\omega$. Thus, one can find the total mechanical moment $J_{\rm abs}=\eta I\pi p^2\tau_{\rm L}/\omega$ absorbed by the cluster and the characteristic moment of an individual electron $M=\gamma m_{\rm e}p^2\omega=J_{\rm abs}/N_{\rm e}=\eta I\pi p^2\tau_{\rm L}/(N_{\rm e}\omega)$. The latter relation makes it possible to estimate the number of electrons in the cluster shell $N_{\rm e}\approx\eta I\pi\tau_{\rm L}/(\gamma m_{\rm e}\omega^2)$, the total charge of the cluster $Q=|e|N_{\rm e}$, and the characteristic density of electrons $n_{\rm e}=3N_{\rm e}/[4\pi(p^3-R^3)]$, i.e., the parameters used in the Lagrange function (2).

The total magnetic moment of the cluster is estimated through its total mechanical moment $N_e \gamma m_e p^2 \omega$ and the gyromagnetic ratio of a relativistic electron $|e|/(2\gamma m_e c)$, where $\gamma = \gamma_L = \sqrt{1+a^2}$ is the characteristic Lorentz factor of a hot electron in a laser target, and a is defined as $a = |e|E_L/(m_e \omega c) = \sqrt{I\lambda^2/W_0}$, $W_0 = 1.37\times10^{18}$ μm^2 cm^{-2}. As a result, the total magnetic moment of the cluster is

$$\mu \approx \frac{|e|}{8m_e\omega\gamma_L}\eta E_L^2 p^2 \tau_L. \tag{11}$$

Outside the cluster, the spatial configuration of the magnetic field corresponds to the magnetic dipole field $\boldsymbol{H}(\boldsymbol{r}) = -\mu \boldsymbol{e}_x/r^3 + 3\mu x\boldsymbol{r}/r^5$; inside the cluster, the magnetic field is uniform. The maximum value of the magnetic field H_{max} at the end of the laser pulse is estimated as the field on the x axis of the dipole (in this case, the field is the same at the centre and at the poles):

$$H_{max}(\tau_L) = \boldsymbol{e}_x \boldsymbol{H}(p\boldsymbol{e}_x) = \frac{2\mu}{p^3} = \frac{|e|\eta E_L^2\tau_L}{4m_e\omega\gamma_L p},$$
$$\frac{H_{max}(\tau_L)}{E_L} = \frac{|e|\eta E_L\tau_L}{4m_e\omega\gamma_L p} = \eta\frac{a}{\sqrt{1+a^2}}\frac{c\tau_L}{4p}. \tag{12}$$

Comparison of the estimate of $H_{max}(\tau_L)$ by formula (12) with the result of numerical simulation [17] is shown in Fig. 2. Note that according to (12), $H_{max}(\tau_L) \sim \tau_L$. This means that the maximum magnetic field increases linearly with time during the laser pulse: $H_{max}(t) = H_{max}(\tau_L)t/\tau_L$, and $t \leqslant \tau_L$. Based on (12), one can also predict a stronger field for a cluster with a smaller radius: $H_{max}(\tau_L) \sim p^{-1} \sim R^{-1}$ (in the absence of a Coulomb explosion of the cluster), which agrees with the results of calculations in [17]. Since the magnetic field of the dipole decreases in space as r^{-3}, the average magnetic field in the focal volume with the cluster density n_{cl} after the end of the laser pulse can be estimated by the formula

$$\langle H\rangle \approx H_{max}(\tau_L)\left(\frac{p}{n_{cl}^{-1/3}}\right)^3 = H_{max}(\tau_L) n_{cl} p^3, \quad n_{cl}p^3 < 1. \tag{13}$$

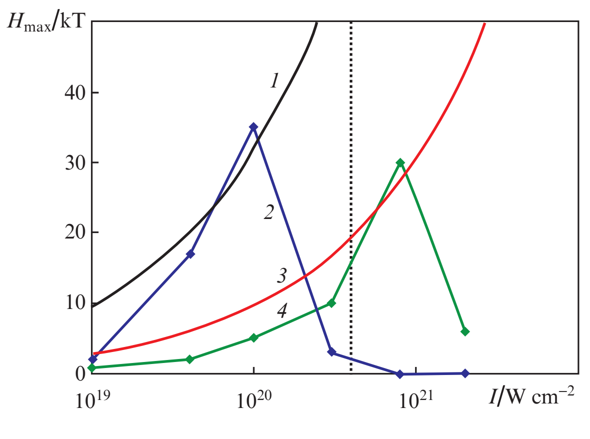

Figure 2. Maximum magnetic field of a Xe^{+20} cluster (initial radius, $R = 50$; electron density, $182\,n_{cr}$) irradiated by a laser pulse with a duration of [(*2*), PIC calculation] 10 and [(*4*), PIC calculation] 3 fs; curves (*1*) and (*3*) are plotted according to formula (12) with $\eta = 0.15$ and $p = 4R$. The vertical dashed line shows the laser intensity corresponding to the Coulomb explosion of the cluster (removal of all electrons).

The field with the amplitude $H_{max}(\tau_L)$ exists during the cluster lifetime τ_{cl}; therefore, estimates using (11) and (12) are valid for short laser pulses: $\tau_L < \tau_{cl}$ (the cluster lifetime τ_{cl} is estimated below). Note that formulae (11) and (12) for the average values of the magnetic moment and the field are limited by the laser field amplitude: $a \leqslant a_{tr}$, $a_{tr} = 2Ze^2 n_i R\lambda/(3m_e c^2)$ (n_i is the initial density of cluster ions), corresponding to the absence of a Coulomb explosion of the cluster (detachment of all electrons by the laser field). In Fig. 2, the laser intensity corresponding to a_{tr} is shown with a vertical dashed line. For a very short pulse (3 fs), the magnetic field does not disappear at $a \to a_{tr}$, which is due to the incompleteness of transient processes (electron acceleration) at such a short pulse duration. For a longer pulse (10 fs), the magnetic field disappears at $a \to a_{tr}$. Note that in the estimates of the magnetic field in our published work [18], we assumed that the laser pulse is sufficiently short ($c\tau_L/4p \sim 1$) and there is no dependence of the magnetic field strength on the pulse duration and cluster radius.

3. Dynamics of the magnetic field of the cluster during the laser pulse action

In addition to estimating the characteristic value of the electron angular momentum $M(\tau_L)$ after the end of the laser pulse, Eqns (4) and (9) make it possible to assess the dependence of the electron angular momentum $M(t) = \gamma(t)m_e r^2\dot{\alpha}(t)$ on time: $M(t) \approx M(\tau_L)t/\tau_L + \delta M(t)$, where the oscillating addition associated with the action of an alternating laser field is determined, taking into account (4), by the equation

$$\frac{d\delta M(t)}{dt} = eE_L p[1 - \delta\dot{x}(t)/c]\sin(k\delta x(t))$$
$$\sim eE_L pk\delta x(t). \tag{14}$$

The amplitude of longitudinal oscillations, $\delta x(t)$, in formula (14) is determined by equation (9).

It can be seen from (14) that during the laser pulse, there occur oscillations of the mechanical moment of the electron, $\delta M(t) \approx \delta x(t)|e|E_L pk/\omega_l$, relative to the characteristic (average over the laser pulse period) value of the moment $M(\tau_L)t/\tau_L$ with frequency

$$\omega_l = \sqrt{\frac{|e|Qk^2}{pm_e\gamma_L}\left(\frac{3}{10} + \frac{1}{kp} + \frac{E_L p^2}{2Q}\right)}$$

of longitudinal oscillations of an electron in the cluster shell.

As shown above, when an electron moves in the Coulomb field of a cluster, the potential energy is $|e|Q/p \approx \gamma m_e c^2$. Accordingly, the frequency ω_l differs from the laser frequency ω by a factor of the order of unity. The maximum magnetic field of the cluster [similar to the mechanical moment of the cluster $N_e M(t)$] during the laser pulse action has an average (over the laser period) component $H_{max}(t) = H_{max}(\tau_L)t/\tau_L, t \leqslant \tau_L$ linearly increasing in time and a variable component $\delta H_{max}(t) = 2|e|N_e\times\delta M(t)/(\gamma_L m_e c p^3)$ oscillating with a frequency close to the laser frequency. Such a time dependence of the x-component of the magnetic field (linearly increasing average value and oscillations against its background) during the laser pulse is confirmed by numerical simulation [17]. Figure 3 shows a comparison of the average, linearly increasing in time, component

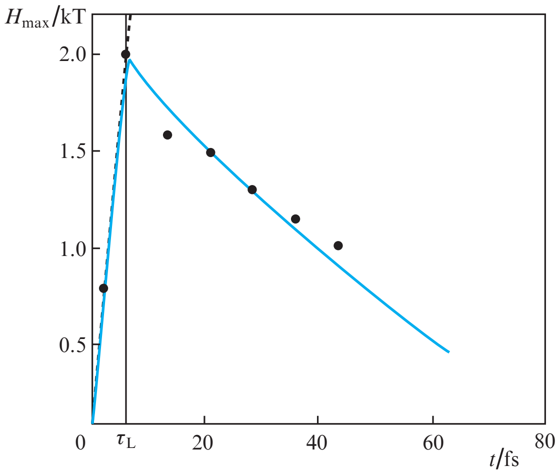

Figure 3. Time dependence of the magnetic field maximum in space, irradiated by a circularly polarsed laser pulse with an intensity of 10^{20} W cm^{-2} (a = 5.2) and a duration of 6 fs for a Xe^{+20} cluster with an initial radius of 50 nm and a density $n_e = 182 n_{cr}$ [points are the results of numerical simulation [17], and the solid line is the calculation by formula (16)].

$$H_{max}(t) = H_{max}(\tau_L)t/\tau_L = \frac{|e|\eta E_L^2 t}{4m_e\omega\gamma_L p}$$

with the results of numerical calculations [17] in a time interval $[0, \tau_L]$ for τ_L = 6 fs. It can be seen that the time-linear increase in the maximum magnetic field value during the laser pulse action corresponds to the results of numerical simulation.

4. Assessment of the nanocluster lifetime and dynamics of the magnetic field after the end of the laser pulse

Dissipative forces act on moving electrons in the cluster shell, and its ionic core scatters in space. Due to the removal of some of the electrons, the ion core acquires a charge and is subjected to the action of Coulomb forces. The outer electron shell also attracts ions, causing the cluster to fly apart. As a result, there are three forces in the hydrodynamic equation for the ion velocity $v_i(r,t)$, which cause expansion:

$$m_i\frac{\partial v_i}{\partial t} + m_i v_i\frac{\partial v_i}{\partial r} = -\frac{1}{n_i}\frac{\partial}{\partial r}\left[m_e c^2(\gamma_L - 1)n_e + \frac{\boldsymbol{H}^2}{8\pi}\right]$$

$$+\frac{4\pi}{3}Ze^2(Zn_i - n_e)r, \qquad (15)$$

where Z and m_i are the charge and mass of the ion. As follows from (15), under the action of the thermal pressure of the electrons, the expansion occurs with a characteristic velocity $v_i \sim v_s \sim c\sqrt{Zm_e(\gamma_L - 1)/m_i}$. The characteristic expansion velocity under the action of the internal Coulomb forces of the ionic nucleus is $v_i \sim v_Q \sim \sqrt{|e|Q/Rm_i}$. The magnetic component of pressure in (15) can be neglected, since the energy density of thermal electrons exceeds the energy density of the magnetic field. The expansion of the cluster core was not taken into account in equations (4)–(9) of the electron motion of the cluster shell. Estimates of the value inverse to the frequency of Coulomb collisions of hot electrons (lifetime) show that it is significantly greater than R/v_s and R/v_Q, i.e. the times of cluster expansion under the action of forces in (15). The radiation losses of rotating electrons are also small. Cyclotron radiation power emitted by electrons is

$$P_{rad} = N_e\frac{2e^4 H_{max}^2(\tau_L)\gamma_L^2}{3m_e^2 c^3} = \gamma_L^2 N_e\frac{32\pi^2 e^6\eta^2 I^2}{27m_e^4 c^7\omega^2}.$$

The ratio of the emitted power to the laser power absorbed by the cluster (electron energy) has the form

$$\frac{P_{rad}}{\eta\pi p^2 I} = \gamma_L^2 N_e\frac{32\pi e^6\eta I}{27m_e^4 c^7\omega^2 p^2} = \gamma_L^2 N_e\frac{8\eta a^2 r_0^2}{27p^2} \ll 1,$$

$$r_0 = \frac{e^2}{m_e c^2}.$$

Since $r_0^2/p^2 \sim 10^{-16}$, the inequality is fulfilled for any reasonable parameters of the laser pulse and the cluster size. Thus, the main channel of dissipation (cooling of electrons) is the expansion of the ionic core of the cluster and an increase in its radius (slow in comparison with the 'period' of electron rotation).

At a laser intensity far from the threshold value of the Coulomb explosion ($a \ll a_{tr}$), due to the azimuthal symmetry of the quasi-stationary fields of the cluster after the end of the laser pulse, the angular momentum of the electron is conserved and is an adiabatic invariant

$$I_\alpha = \frac{1}{2\pi}\int_0^{2\pi}\frac{\partial L}{\partial\dot\alpha}d\alpha \approx M(\tau_L) = \text{const}(t),$$

which is not affected by a slow change in the cluster parameters. In this case, the orbital magnetic moment of the electron μ depends on time, since the electron energy decreases during the expansion of the cluster: $\mu(t) = |e|M(\tau_L)/(2\gamma(t)m_e c)$. The magnetic field of an electron ball (cluster shell) rotating and expanding in space is determined by the magnetisation vector (magnetic moment per unit volume) depending on the electron density, which also decreases during expansion. As a result, we have the following dependence of the maximum magnetic field (field on the x axis of the cluster) on time after the end of the laser pulse:

$$H_{max}(t) \approx H_{max}(\tau_L)\frac{n_e(t)\gamma_L}{n_e(\tau_L)\gamma(t)}, \quad t > \tau_L,$$

where $n_e(\tau_L) = n_e$ and γ_L are defined above. Let the radius of the ionic core of the cluster increase as $R(t)$. Then $n_e(t) \sim R^{-3}(t)$, $\gamma(t) = 1 + (\gamma_L - 1)[R(0)/R(t)]^2$. For the time dependence of the x-component of the magnetic field averaged over the laser period during and after the end of the laser pulse, the following formula is valid:

$$H_{max}(t) \approx \frac{|e|\eta E_L^2\tau_L}{4m_e\omega\gamma_L p}\Big\{(t/\tau_L)\theta(1 - t/\tau_L) + \theta(t/\tau_L - 1)$$

$$\times\frac{R^3(0)\gamma_L}{R^3(t)[1 + (\gamma_L - 1)(R(0)/R(t))^2]}\Big\}, \qquad (16)$$

where $\theta(t)$ is the Heaviside step function. Recall that during the laser pulse action, there is also an oscillating part of the x-component of the field, $\delta H_{max}(t)$.

In the case of strongly relativistic electrons, $H(t) \sim R^{-1}(t)$; for nonrelativistic electrons, $H(t) \sim R^{-3}(t)$. The time dependence of the cluster radius $R(t)$ is determined by the expansion regime (Coulomb or thermal) of the cluster:

$$\frac{R(t)}{R(0)} \approx 1 + \frac{v_s t}{p}, \quad Zm_e c^2(\gamma_L - 1) > Z|e|Q/R(0),$$

$$\begin{cases} R(t) = R(0)\mathrm{ch}^2\xi, \\ t = \dfrac{\xi + 0.5\mathrm{sh}(2\xi)}{2\sqrt{2|e|Q/(m_i R^3(0))}}, \quad \xi \in [0, \infty], \\ Zm_e c^2(\gamma_L - 1) < Z|e|Q/R(0). \end{cases} \tag{17}$$

Field (16) increases linearly during the laser pulse, remains approximately constant during the cluster lifetime $\tau_{cl} \approx p/v_{s,Q}$, and then decreases $\sim t^{-1}$ for relativistic clusters or $\sim t^{-3}$ for nonrelativistic ones. Figure 3 shows the time dependence of the quasi-stationary magnetic field $H_{max}(t)$ (solid curve) upon irradiation with a circularly polarised laser pulse with an intensity of 10^{20} W cm^{-2} ($a = 5.2$) and a duration of 6 fs for an Xe cluster with an initial radius of 50 nm, a density $n_e = 182 n_{cr}$ and xenon ionisation rate $Z = 20$; the dots show the data of the PIC calculation [17]. One can see the coincidence of the time dependence (16) with the results of numerical simulation during the action of the laser pulse and after its end.

5. Conclusions

An analytical model is proposed for the generation of a large-scale (tens of microns) magnetic field with an intensity of up to several GG, which exists during a picosecond time interval in the focal waist of an ultra-high-power short laser pulse. This field is formed by relativistic magnetic dipoles arising in a gas of neutral nanoclusters under irradiation with a short (tens of fs), relativistically intense, circularly polarised laser pulse. In comparison with our previous works, we have constructed in this study an analytical model of the generation of a magnetic field, which makes it possible to take into account the dependence of the field strength on the cluster radius and the duration of the laser pulse, to study the temporal dynamics of the magnetic field during a laser pulse, and also to find the lifetime and asymptotics of the dependence of the field on time at large (in comparison with the laser pulse duration) time intervals. Note that the amplitude of the quasi-stationary field reaches tenths of the laser field amplitude, and the field lifetime is determined by the nanocluster expansion time and is significantly (tens of times) longer than the laser pulse duration. The slow decay of the magnetic field in time ($\sim t^{-1}$) allows maintaining a magnetic field of $\sim$1 GG at picosecond time intervals. Implementation of a cluster laser target will allow gigagauss magnetic fields to be generated under laboratory conditions and the properties of a highly magnetised laser plasma to be experimentally studied. Such studies can be relevant for astrophysical applications; for example, they make it possible to experimentally simulate plasma in the vicinity of neutron stars with a superstrong magnetic field.

Appendix. Absorption of circularly polarised laser radiation by a nanocluster

When the electron cloud of the cluster is displaced relative to the ion core, a returning ambipolar field appears between the electron shell and the ion core

$$\boldsymbol{E} = \frac{4\pi e n_e}{3}\boldsymbol{r};$$

as a result, the equation of motion of the electron shell takes the form (nonrelativistic case)

$$m_e \dot{\boldsymbol{r}} = -\frac{1}{3} m_e \omega_{pe}^2 \boldsymbol{r} - m_e v_{ei} \dot{\boldsymbol{r}}$$

$$+ eE_0[\boldsymbol{e}_x \cos(kz - \omega t) + \boldsymbol{e}_y \sin(kz - \omega t)], \tag{A1}$$

where ω_{pe} is the plasma frequency of electrons, and v_{ei} is the frequency of electron–ion collisions.

The solution to this equation out of resonance in the zeroth approximation in kz leads only to collisional absorption with the coefficient [21]

$$\eta_{ei} = \frac{9}{2} \frac{v_{ei}\omega^2\omega_{pe}}{2[\omega_{pe}^2(\omega_{pe}^2 - 6\omega^2) + 9\omega^2(\omega^2 + v_{ei}^2)]}.$$

Collisional absorption is relevant at the initial stage of interaction, when the electron temperature is not too high. When the cluster is heated to temperatures

$$T_e \approx mc^2\left(\sqrt{1 + \frac{I\lambda^2}{W_0}} - 1\right),$$

where $W_0 = 1.37 \times 10^{18}$ W μm^2 cm^{-2}, the collision frequency decreases rapidly and the absorption becomes small. In particular, at $n_e = Zn_i = 8 \cdot 6 \times 10^{22}$ cm^{-3} (water cluster) and an intensity of 10^{17} W cm^{-2} collisional absorption is 5×10^{-6}; therefore, the main mechanism is collisionless resonance absorption.

The solution to equation (A1) at $v_{ei} = 0$ has the form

$$\boldsymbol{r}(t) = \frac{eE_0\boldsymbol{e}_x}{\omega_{pe}^2/3 - \omega^2}[\cos\omega t - \cos(\omega_{pe}t/\sqrt{3})]$$

$$-\frac{eE_0\boldsymbol{e}_y}{\omega_{pe}^2/3 - \omega^2}[\sin\omega t - \sin(\omega_{pe}t/\sqrt{3})]. \tag{A2}$$

Equation (A2) contains a solution to the homogeneous equation for the subsequent disclosure of the uncertainty at the resonance $\omega \to \omega_{pe}/\sqrt{3}$. Outside resonance, solution (A2) does not lead to absorption, since $\langle \boldsymbol{E}\dot{\boldsymbol{r}} \rangle_t = 0$. However, as $\omega \to \omega_{pe}/\sqrt{3}$, it grows linearly with time,

$$\boldsymbol{r}(t) = \frac{eE_0\boldsymbol{e}_x t}{2\omega_{pe}/\sqrt{3}}\sin(\omega_{pe}t/\sqrt{3})$$

$$+\frac{eE_0\boldsymbol{e}_y t}{2\omega_{pe}/\sqrt{3}}\cos(\omega_{pe}t/\sqrt{3}),$$

which corresponds to motion in a circle with a linearly increasing radius, i.e. to resonance with circular polarisation.

Let us find the power absorbed per unit volume:

$$\frac{\mathrm{d}^2U}{\mathrm{d}t\mathrm{d}V} = <|e|n_{\mathrm{e}}\dot{\boldsymbol{r}}\boldsymbol{E}>_t = \frac{e^2n_{\mathrm{e}}E_0^2}{2\omega}[(\sin\omega t$$

$$+\omega t\cos\omega t)\cos\omega t - (\cos\omega t - \omega t\sin\omega t)\sin\omega t]$$

$$= \frac{e^2n_{\mathrm{e}}E_0^2t}{2},\ \omega = \omega_{\mathrm{pe}}/\sqrt{3}.$$

Absorbed energy density (integral of power over time) has the form

$$\frac{\mathrm{d}U}{\mathrm{d}V} = \frac{e^2n_{\mathrm{e}}E_0^2t^2}{4} = \frac{E_0^2}{16\pi}\omega_{\mathrm{pe}}^2t^2 = \frac{3E_0^2}{16\pi}\omega^2t^2.$$

This formula coincides with formula (12) in [20], where linearly polarised radiation was considered, up to a numerical factor. The resonance condition $\omega = \omega_{\mathrm{pe}}/\sqrt{3}$ in the shell of the cluster can be fulfilled because:

1) in a real laser field of a time-limited pulse, $E_0(t) = E_0\exp(-t^2/\tau_{\mathrm{L}}^2)$, in the Fourier spectrum contains a resonance frequency;

2) allowance for relativistic corrections will lead to the replacement $\omega \to \omega_{\mathrm{pe}}/\sqrt{\gamma_{\mathrm{L}}}$, and the condition $\omega = \omega_{\mathrm{pe}}/\sqrt{3\gamma_{\mathrm{L}}}$ is more favourable for resonance;

3) expansion of the cluster begins already during the action of the laser pulse; therefore, the ratio $\omega_{\mathrm{pe}}/\omega \sim 10$, which is valid for 'solid-state' values of the electron density in the cluster shell, becomes less than 10; and

4) when the electron density decreases with increasing radius, there is always a resonance point at which the condition $\omega = \omega_{\mathrm{pe}}/\sqrt{3\gamma_{\mathrm{L}}}$ is satisfied locally. Thus, in the case of circular polarisation, the main mechanism of cluster absorption is resonant collisionless absorption, and the absorption coefficient is determined by the expression

$$\eta_{\mathrm{r}} = \eta = \frac{4\pi R^2 l_{\mathrm{s}}\,\mathrm{d}U/\mathrm{d}V}{\pi R^2\tau_{\mathrm{L}}\,cE_{\mathrm{L}}^2/4\pi},$$

where l_{s} is the thickness of the skin layer.

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