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Controlling logical operations with images in accumulated echo holography using the effects of information locking and erasing

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Abstract. **The possibility of implementing logical operations with images using the accumulated echo hologram in the presence of external spatially inhomogeneous electric fields is considered. It is shown that the photon echo locking effect makes it possible to control the execution of logical operations (from the union of sets to the symmetric difference and their superposition), as well to control their form by varying the values of the gradients of external spatially inhomogeneous electric fields and the phase difference between pairs of pulses.**

Keywords: *logical operations, echo holography, locking efficiency, symmetric difference, union of sets.*

1. Introduction

Recently, the development of quantum memory [1, 2] with the use of a photon echo has become very relevant [3]. The processing of quantum information is proposed to be performed using logical quantum gates. For example, in work [4], an optical scheme for implementing a quantum gate of a generalised controlled phase was proposed, based on the use of nonresonant interaction of photon with a three-level atom in a high-Q resonator. The advantages of the proposed protocol and possible options for its experimental implementation are discussed.

Coherent optical methods for processing information embedded in images have become the most popular, which is due to the capability of filtering and transforming images in the nanosecond domain. Of particular interest is the echo-holographic processor (EHP), which belongs to the class of multifunctional analogue devices. Due to the presence of control signals, its impulse response can be programmed in real time and receive various types of processing – from simple memorisation to integral transformations [5].

Optical echo holography makes it possible to implement logical operations with images. In work [6], logical operations on sets represented as images (the intersection of sets) are considered using a stimulated echo hologram. In work [7], the operation of uniting sets represented in the form of images is performed using the accumulated long-lived echo hologram (ALLEH) regime. It is shown in work [8] that, depending on the phase difference between pairs of exciting laser pulses in the ALLEH, it is possible to perform logical operations of uniting sets, constructing symmetric difference of sets, and also superposing these operations.

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In this paper, we consider a number of logical operations on images using the ALLEH in the presence of a phase difference between pairs of exciting laser pulses. To control these operations, the most promising is the effect of locking echo holographic information, which means the formation of conditions under which the recorded information cannot appear (or appears only partially) in the form of a response of the resonant medium, which can be achieved by violating the frequency-temporal correlation of the inhomogeneous broadening of the resonant lines at different time intervals. This frequency-temporal correlation is related to the strict correspondence of individual so-called monochromates of the inhomogeneously broadened line at different time intervals. Each monochromate is formed by a set of atoms (molecules and ions) under the same conditions (for example, in local fields in crystals), but randomly distributed over the sample volume. A set of identical impurity centres located in one or several equal local fields is characterised, in an inhomogeneously broadened line, by a certain spectral region, which is called a spin packet, or monochromate. The spectral width of a monochromate is inversely proportional to the time of transverse irreversible relaxation of the system, whereas the inhomogeneously broadened line width is inversely proportional to the time of reversible relaxation. In crystal lattices, there are always inhomogeneities, random dislocations, mechanical stresses, etc.; therefore, each impurity centre is located in its own local crystal field and has its own optical transition frequency. In gases, the cause of the frequency difference is the Doppler effect. As a result, the observed radiation line is a superposition of a large number of uniformly broadened lines (monochromates), each of them having its own frequency.

The process of forming photon echo responses consists of two main stages: dephasing of the oscillating dipole moments of the optical centres and their subsequent phasing, which leads to the appearance of macroscopic polarisation of the medium, observed in the form of a specific response. Therefore, even a slight violation of the strict frequency-temporal correlation of inhomogeneous broadening should lead to a significant weakening of the response intensity.

In other words, we are talking about the reversible destruction of the phase memory of the resonant medium with the possibility of its restoration. This effect can be achieved by exposing the resonant medium to various spatially inhomogeneous external perturbations at different time intervals, resulting in random shifts or splits of the original monochromates of the inhomogeneously broadened line.

Note that in work [9], the effect of locking the long-lived photon echo (LLPE) in the $LaF_3:Pr^{3+}$ crystal ($^3H_4-^3P_0$, transition, $\lambda = 477.7$ nm) under the impact of a spatially inhomogeneous electric field in the interval between the first and second laser pulses was theoretically predicted and experimentally confirmed. In work [9], the efficiency of suppressing the response of stimulated photon echo (SPE) was investigated for various schemes of exposure of the resonant medium to spatially inhomogeneous electric fields.

Since the ALLEH response is a superposition of the LLPE responses from N pairs of exciting pulses and a readout pulse, the contribution to this response from each pair of exciting pulses becomes different (it depends on the magnitude of the gradients of external spatially inhomogeneous electric fields and their mutual orientation, as well as on the phase difference between the pairs of exciting laser pulses). It is shown in works [10, 11] that the formation of a phase difference between pairs of exciting pulses can lead to a decrease or disappearance of the frequency modulations of populations during the response formation, which leads to its disappearance. Thus, if the phases between the pairs of exciting pulses differ from each other by π, the ALLEH signal intensity is significantly reduced for image elements being identical on the transparencies in the first and second, as well as in the second and third pairs of exciting pulses. This makes it possible to control logical operations with the use of ALLEH.

2. Basic equations

We consider the efficiency of locking and reproducing images in the ALLEH regime (three pairs of exciting pulses shifted in phase relative to each other) under the impact of external spatially inhomogeneous electric fields in the time intervals between exciting resonant laser pulses in each pair (Fig. 1). We assume that every second laser pulse in the pair passes through the corresponding transparency with the image.

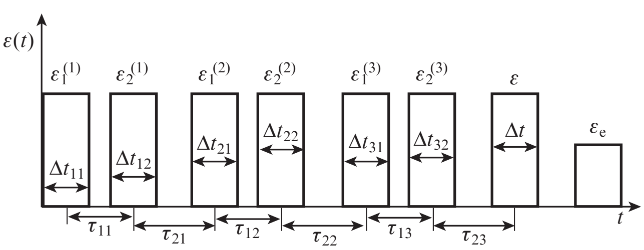

Figure 1. Sequence of exciting laser pulses in the formation of ALLEH responses: $\Delta t_{1i} = \Delta t_{2i} = \Delta t_{3i} = \Delta t$ $(i = 1, 2)$, $\tau_{11} = \tau_{12} = \tau_{13} = \tau_1$, $\tau_2 \approx \tau_{21} + \tau_{22} + \tau_{23} + 2\tau_1$, $\tau_1 \ll T_2$, and $\#\tau_2 \ll T_1$ (T_1 and T_2 are the times of longitudinal and irreversible transverse relaxation of the resonant system, $j = 1, 2, 3$ is the number of a pair of exciting pulses, $\varepsilon_n^{(j)}$ is the amplitude of the nth pulse in the ith pair of exciting pulses, ε is the amplitude of the readout pulse, and ε_e is the amplitude of the ALLEH response).

In work [10], a method for erasing information was proposed. The SPE response is formed in such a way that after two-pulse laser excitation, information is transferred to the population lattice. To destroy the recorded information, a similar pair of pulses is used, but with a phase shift of 180°. This leads to the formation of a phase-shifted population lattice; therefore, when two population lattices are superimposed, they compensate for each other, which leads to a decrease in the response intensity after the readout pulse action.

In work [10], it was proposed to pass excitation pulses through an electro-optical crystal with transparent electrodes to obtain a phase shift, which allows controlling the pulse phase. During the recording of information, there is no voltage difference between the crystal electrodes, which results in a certain phase difference between the pulses. If it is necessary to erase information (to reduce the ALLEH response intensity), a certain electric voltage must be applied to the crystal, which results in an additional phase difference between the pulses.

We can represent the electric field intensity of the ηth exciting laser pulse passed through the corresponding transparency with an image in the form

$$E_\eta(\boldsymbol{r}, t) = U_\eta(\boldsymbol{r})\mathrm{e}^{\mathrm{i}\omega t} + \text{c.c.}, \quad 0 \leqslant t \leqslant \Delta t_\eta, \tag{1}$$

where Δt_η is the duration of the ηth exciting laser pulse, and $U_\eta(\boldsymbol{r})$ describes its spatial structure.

We consider an image on the transparency as a set of n points with radius vectors $\boldsymbol{r}_n$. Each such point emits a spherical wave. The set of waves at the location of the ith optical centre in the sample with the radius vector $\boldsymbol{r}_{0i}$ determines the magnitude of the perturbation of the resonant transition of the optical centre. Then the electric field intensity of the object laser pulse containing the information (the object is the first pulse of each pair of exciting pulses) at pointr $\boldsymbol{r}_{0i}$ can be written in the form of an expansion in spherical waves:

$$E_t = \sum_n A_{ni} \frac{\exp[\mathrm{i}\boldsymbol{k}_n(\boldsymbol{r}_{0i} - \boldsymbol{r}_n) - \mathrm{i}\omega t + \mathrm{i}\varphi_n]}{|\boldsymbol{r}_{0i} - \boldsymbol{r}_n|}, \tag{2}$$

where

$$\boldsymbol{k}_n = \frac{\omega}{c}\boldsymbol{n}_n; \;\; \boldsymbol{n}_n = \frac{\boldsymbol{r}_{0i} - \boldsymbol{r}_n}{|\boldsymbol{r}_{0i} - \boldsymbol{r}_n|};$$

and φ_n are the initial phases of spherical waves; $\mathrm{e}^{\mathrm{i}\varphi_n}$ can be included in the complex amplitudes A_{ni}. If $|\boldsymbol{r}_{0i} - \boldsymbol{r}_n|$ is much larger than the sample size, then expansion (2) in spherical waves turns into the expansion in plane waves:

$$E_i = \sum_n \varepsilon_n \exp(\mathrm{i}\boldsymbol{k}_n\boldsymbol{r}_{0i} - \mathrm{i}\omega t), \tag{3}$$

where ε_n are the electric field intensity amplitudes of the plane waves from individual points of the object. Since one of each pair of exciting laser pulses is an image carrier, the spatial phase matching in the formation of the ALLEH response is described by the expression

$$\boldsymbol{k}_{\mathrm{e}n}^{(j)} = -\boldsymbol{k}_{1n'}^{(j)} + \boldsymbol{k}_{2n''}^{(j)} + \boldsymbol{k}_{3n'''}^{(j)}, \tag{4}$$

where $\boldsymbol{k}_{in}^{(j)}$ are the wave vectors of the plane waves in the spatial expansion of the wavefronts of the object laser pulses for each jth pair; $\boldsymbol{k}_{en}^{(j)}$ is the wave vector of the ALLEH response; j is the number of the pair of pulses; subscripts 1 and 2 are the numbers of exciting pulses in the pair; subscript 3 is the number of the readout pulse; and subscripts n', n'', and n''' determine the sets of plane waves of the spatial expansion of the fields of the object (exciting) pulses after their passage through the corresponding transparencies marked with strokes.

It should be noted that in the absence of external spatially inhomogeneous electric fields, only those components of the response field expansion appear in the ALLEH response, for which the amplitudes of the expansion of the fields of the exciting pulses, corresponding to the directions of the wave vectors, are nonzero. For the formation of a sufficient set of plane (spherical) waves necessary for the implementation of spatial phase matching, pulses that do not carry images must be formed using matte transparencies.

As in works [7, 12], the spatial structure of the ALLEH response is determined by the expression

$$E_{\mathrm{APE}} \sim \sum_{j=1}^{n} E_j(t, \boldsymbol{R}) \exp(\mathrm{i}\Delta\varphi_j), \tag{5}$$

$$E_j \approx \frac{1}{V} \sum_{n',n'',n'''} \int_V \mathrm{d}V \int_{-\infty}^{\infty} g(\Delta)\,\mathrm{d}\Delta \sin\theta_1^{(j)} \sin\theta_2^{(j)} \theta_3$$

$$\times \varepsilon_{1n'}^{*(j)} \varepsilon_{2n''}^{(j)} \varepsilon_{3n'''} \Bigg[\Big| \sum_{n'} \varepsilon_{1n'}^{*(j)} \exp(-\mathrm{i}\boldsymbol{k}_{1n'}^{(j)} \boldsymbol{r}) \Big|$$

$$\times \Big| \sum_{n''} \varepsilon_{2n''}^{(j)} \exp(\mathrm{i}\boldsymbol{k}_{2n''}^{(j)} \boldsymbol{r}) \Big| \Big| \sum_{n'''} \varepsilon_{3n'''} \exp(-\mathrm{i}\boldsymbol{k}_{3n'''}^{(j)} \boldsymbol{r}) \Big| \Bigg]^{-1}$$

$$\times \exp[-\mathrm{i}(\boldsymbol{k}_{0n}^{(j)} + \boldsymbol{k}_{1n'}^{(j)} - \boldsymbol{k}_{2n''}^{(j)} - \boldsymbol{k}_{3n'''}^{(j)}) \boldsymbol{r}]$$

$$\times \exp\{\mathrm{i}[\tau_1 f_j(\Delta, \boldsymbol{r}) - (t - \tau_1 - \tau_2) f(\Delta, \boldsymbol{r})]\},$$

where $\theta_1^{(j)}$, and $\theta_2^{(j)}$ are the areas of the first and second pulses in the jth pair, respectively; θ_3 is the readout pulse area; V is the volume of the excited part of the sample; $g(\Delta)$ is the distribution of optical centres in frequencies; $\Delta = \omega - \Omega_0$; ω is the laser radiation frequency; Ω_0 is the resonant transition frequency; $\varepsilon_{in}^{(j)}$ are the electric field intensity amplitudes of plane waves of the spatial expansion of the wavefronts of the object laser pulses in each jth pair; $\Delta\varphi_j$ is the phase difference between the first and jth pairs of exciting laser pulses; $f_j(\Delta, \boldsymbol{r}) = \Delta + \chi_j(\boldsymbol{r})$; $\chi_j(\boldsymbol{r})$ is the additional frequency shift of the optical centre transition frequency in the time interval τ_{1j} between pulses in the jth pair, which occurs due to an external inhomogeneous electric field; and $f_j(\Delta, \boldsymbol{r})$ is the frequency shift caused by the action of a local field in the crystal and an external inhomogeneous electric field. In the case of the linear Stark effect we have $\chi(\boldsymbol{r}) = C_{\mathrm{Sh}}(E_0 + \nabla E\boldsymbol{r})$, while in the case of the quadratic Stark effect, $\chi(\boldsymbol{r}) = C'_{\mathrm{Sh}}(E_0 + \nabla E\boldsymbol{r})^2$, where C_{Sh}, and C'_{Sh} are the Stark coefficients; and E_0 is the external inhomogeneous electric field intensity at $\boldsymbol{r} = 0$.

It was shown in work [13] that the formation of a phase difference between pairs of exciting pulses can lead to a decrease or disappearance of the frequency modulations of populations during the response formation, which leads to its disappearance. Thus, if the phase differences between the pairs of exciting pulses differ from each other by π, the ALLEH signal intensity is significantly reduced for image elements that are identical on the transparencies in the first and second, second and third pairs of exciting pulses (the effect of erasing information). Varying the phases and magnitudes of the gradients of spatially inhomogeneous electric fields allows controlling the implementation of logical operations with images.

3. Control of logical operations with images by means of spatially inhomogeneous electric fields and phase relationships

Figure 1 shows a sequence of exciting pulses used in the implementation of logical operations of symmetric difference, union of sets, and difference, as well as their superposition, in the presence of external spatially inhomogeneous electric fields. The ALLEH excitation can be carried out in a $LaF_3:Pr^{3+}$ crystal, where the time interval between pulses in pairs can be tens of nanoseconds, while the time intervals between pairs of pulses and between the last recording pair and the readout pulse can reach tens of minutes at helium temperatures of the crystal.

Numerical calculation of the ALLEH response using expression (5) shows that the generated image is a superposition of images contained in exciting laser pulses, depending on the magnitude of the gradients of spatially inhomogeneous electric fields, which leads to the possibility of implementing the corresponding logical operations.

The union of sets A and B is a set consisting of all elements of the original sets. The difference between sets A and B is a set that includes all the elements of the first set which are not included in the second set. The symmetric difference of sets A and B is a set that includes all the elements of the original sets, which do not simultaneously belong to both of the original sets.

To illustrate the logical operations in the ALLEH regime, transparencies with images in the form of an ellipse are considered as sets A, B (Fig. 2).

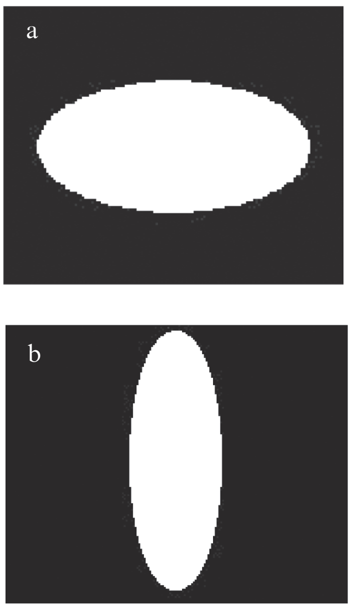

Figure 2. Transparencies: (a) set A and (b) set B.

Figures 3 and 4 show the images obtained in the ALLEH response when a corresponding image is embedded into each pair of pulses (see Fig. 1), and the phase difference between the pairs of exciting laser pulses and the gradients of external spatially inhomogeneous electric fields varies.

Figure 3 shows the case of implementing the operation of uniting sets A and B.

In work [14], it was shown that for $\Delta\varphi_1 = 60°$ and $\Delta\varphi_2 = 180°$, the operation of difference of sets A and B is implemented (Figs 4c and 4f). When exposed to external spatially inhomogeneous electric fields, it is possible to control the type of logical operations. For example, for $\nabla E = \nabla E_1 = \nabla E_3 = 0$, and $\nabla E_2 = 10$ V cm^{-2}, the operation of symmetric difference is implemented (Fig. 4e). In Fig. 4g, only the first transparency is reproduced, although the second transparency was also involved, and this effect is possible when an external electric field with a gradient $\nabla E_3 = 110$ V cm^{-2} is applied in the time interval between the pulses in the third pair of exciting pulses. At a gradient $\nabla E = 130$ V cm^{-2} of the external electric field in the time interval between the readout laser pulse and the ALLEH response, the image in the response disappears (Fig. 4h).

Figure 3. Image in the ALLEH response at $\nabla E_1 = \nabla E_2 = \nabla E_3 = \nabla E = 0$, $\Delta\varphi_1 = 0$, and $\Delta\varphi_2 = 0$.

The obtained responses in the ALLEH regime contain images that are the result of logical operations of uniting sets at zero gradients of external spatially inhomogeneous electric fields, as well as taking the difference and the symmetric difference when exposed to external spatially inhomogeneous electric fields. At certain values of gradients and phases, a superposition of these logical operations is observed.

As an example of the practical application of logical operations with real images, consider images of several localities that do not touch each other, or the same locality at different time moments. Previously, one had to download the actual images separately to track updates to the images covering these localities. Now it is possible to combine images describing all the required localities into a common image and operate with it (download, copy, export, etc.). This will allow us to perform the necessary actions only once, starting the process and tracking the changes within nanoseconds.

Thus, in this paper, we have considered the implementation of logical operations with images using ALLEH in the presence of spatially inhomogeneous electric fields and a nonzero phase difference between pairs of exciting pulses, as well as the control over these operations. It is shown that it is possible to perform a number of logical operations with images on a nanosecond time scale, which will allow for appropriate filtering and image processing.

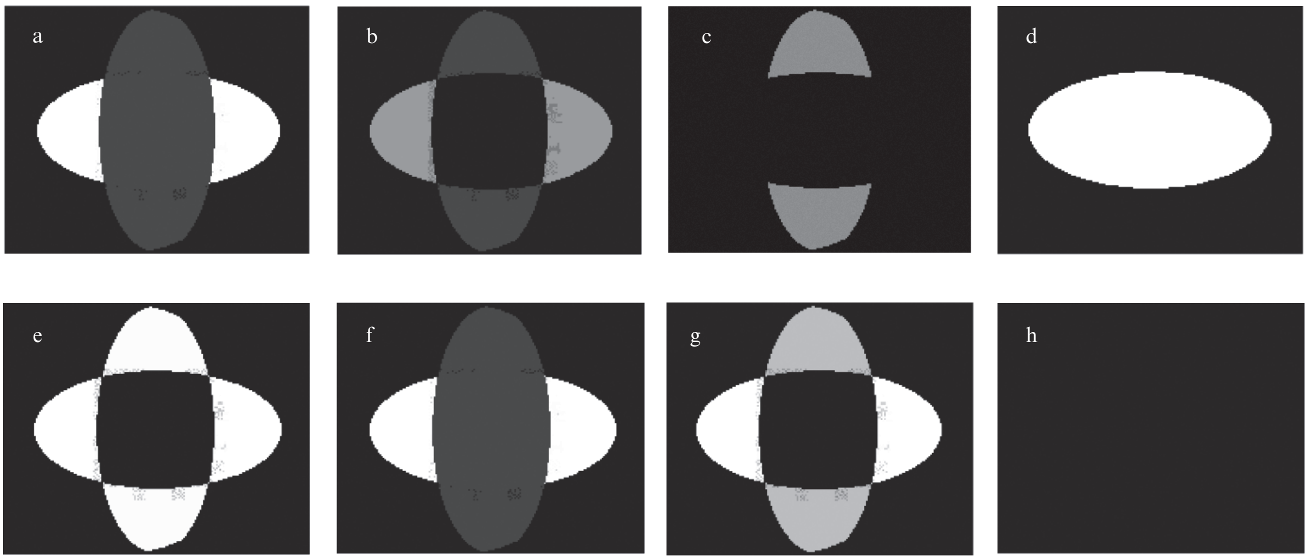

Figure 4. Images in the ALLEH response ($\Delta\varphi_1 = 60°$, $\Delta\varphi_2 = 180°$) at (a) $\nabla E = \nabla E_2 = \nabla E_3 = 0$, $\nabla E_1 = 0$ (superposition of the logical operations of union and difference); (b) $\nabla E = \nabla E_2 = \nabla E_3 = 0$, $\nabla E_1 = 70$ V cm^{-2} (superposition of logical operations of symmetric difference and union); (c) $\nabla E = \nabla E_2 = \nabla E_3 = 0$, $\nabla E_1 = 150$ V cm^{-2} (logical operation of difference); (d) $\nabla E = \nabla E_2 = \nabla E_3 = 0$, $\nabla E_1 = 180$ V cm^{-2} (partial locking of the response); (e) $\nabla E = \nabla E_1 = \nabla E_3 = 0$, $\nabla E_2 = 10$ V cm^{-2} (logical operation of symmetric difference); (f) $\nabla E = \nabla E_1 = \nabla E_3 = 0$, $\nabla E_2 = 140$ V cm^{-2} (logical operation of difference); (g) $\nabla E = \nabla E_1 = \nabla E_2 = 0$, $\nabla E_3 = 110$ V cm^{-2} (logical operation of symmetric difference); (h) $\nabla E_3 = \nabla E_1 = \nabla E_2 = 0$, $\nabla E = 130$ V cm^{-2} (locking of the response).

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