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Zero shift in a Zeeman laser gyroscope with periodic modulation of intracavity losses

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Abstract. **Periodic modulation of intracavity losses is shown theoretically to result in a change in the shape of the alternating frequency dithering created in a Zeeman ring laser when a magnetic field is applied to the active medium. A mechanism is proposed of a vibrational zero shift in a Zeeman laser gyroscope due to modulation of intracavity losses during vibration of the resonator mirrors.**

Keywords: *Zeeman laser gyroscope, frequency dithering, vibrational zero shift, modulation of intracavity losses.*

1. Introduction

Laser gyroscopes (LGs) with frequency dithering based on the Zeeman effect are highly immune to external mechanical influences [1, 2]. In the presence of mechanical shocks and shock-induced vibrations of the mirrors, there occurs an additional zero shift in the LG, called vibrational. The phase nonreciprocity of the ring resonator, which exists at a zero rotation velocity, is called the zero shift in the LG. The instability of this dithering is one of the factors limiting the measurement accuracy. In contrast to LGs with a vibration frequency dithering, using vibration suspension, the body of the optical resonator in Zeeman laser gyroscopes (ZLGs) is rigidly fixed on the base, which significantly reduces the sensitivity to shock. Nevertheless, a decrease in the vibrational zero shift is an important problem for the ZLG as well [3, 4]. The analysis of the zero shift in the ZLG in the absence of external mechanical influences was carried out in [5–7].

To leave the dead zone in the ZLG, an alternating frequency dithering is used, which arises due to the Zeeman effect when a magnetic field is applied to the active medium. The dithering is usually rectangular:

$$\Omega_B(t) = \begin{cases} \Omega_\mathrm{d} \text{ for } 0 < t < T_\mathrm{d}/2, \\ -\Omega_\mathrm{d} \text{ for } T_\mathrm{d}/2 < t < T_\mathrm{d}, \end{cases} \tag{1}$$

where Ω_d and T_d are the amplitude and period of the dithering. The dithering amplitude Ω_d depends on the oscillation frequency detuning ξ relative to the centre of the gain line:

$$\Omega_\mathrm{d} = \Omega_\mathrm{d0}(1 - A\xi^2), \tag{2}$$

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where Ω_d0 is the dithering amplitude at $\xi = 0$; A is the proportionality factor. For simplicity, we will consider a laser with one Ne isotope.

The ZLG has a system for self-tuning the perimeter of the ring resonator, which, in the absence of external mechanical influences, provides a detuning of the generation frequency $\xi = 0$. Under the influence of shocks, the mirrors of the optical resonator vibrate, leading to a change in the perimeter of the optical resonator and to periodic modulation of the detuning of the frequency from the gain line centre:

$$\xi(t) = B\Delta L\sin(2\pi\nu t + \varphi_0). \tag{3}$$

Here ν and φ_0 are the frequency and phase of the sinusoidal vibration of the mirrors; ΔL is the maximum detuning of the resonator perimeter due to the vibration of the mirrors, proportional to the vibration-shock acceleration; and B is the proportionality coefficient. In accordance with (2) and (3), a formula for the vibrational shift of the ZLG zero was obtained in [3].

In the present work, another mechanism leading to vibrational zero shift is theoretically investigated. Due to the deformation of the optical resonator perimeter upon vibration of the mirrors, intracavity losses can modulate, which, as shown below, also leads to distortion of the Zeeman frequency dithering and the appearance of a vibrational zero shift. The aim of this work is to theoretically study the zero dithering in the ZLG with periodic modulation of intracavity losses; a comparison is made of the vibrational shits of zero in the ZLG, introduced by this mechanism and considered in [3, 4].

2. Theory

When studying the ZLG characteristics, we will follow Refs [8, 9]. For the complex amplitudes of counterpropagating waves, we write the system of ordinary differential equations in the form

$$\dot{E}_{1,2} = \frac{\Delta\nu}{2}\Big[\kappa_{1,2}\frac{n}{n_0} - 1 - \delta_\mathrm{v}(t) - \alpha_{1,2}|E_{1,2}|^2 - \beta_{1,2}|E_{2,1}|^2\Big]E_{1,2}, \tag{4}$$

where $\kappa_{1,2}$, $\alpha_{1,2}$, and $\beta_{1,2}$ are coefficients describing the polarisability of the gain medium; n is the relative excess of gain over losses at the maximum of the gain line; n_0 is the value of $\kappa_{1,2}$ at the maximum of the gain line; $\Delta\nu = p_\mathrm{c}/T$ is the resonator bandwidth; p_c is the loss inside the cavity; and T is the round-trip transit time for the light around the resonator perimeter.

Equations (4) take into account the periodic modulation of intracavity losses due to vibration of the mirrors, $\delta_\mathrm{v}(t)$, which we define as

$$\delta_v(t) = \varepsilon \sin(\omega_v t + \varphi_v), \qquad (5)$$

where ω_v and φ_v are the modulation frequency and phase, and εp_c is the amplitude of modulation of intracavity losses. When a magnetic field is applied to a gain medium, a periodic alternating dithering is formed, which has the form of (1). A two-frequency regime is considered, when the fields of counterpropagating waves $E_{1,2}$ correspond to one longitudinal mode and are circularly polarised. Since the amplitude of the frequency dithering is large compared to the width of the lock-in region, the coupling of counterpropagating waves through backscattering is not taken into account in Eqns (4).

2.1. Analytical solution

Taking (4) into account, we obtain a quasi-static solution for the intensities of the counterpropagating waves $|E_{1,2}|^2$ and the frequency dithering $\dot{\Phi}$ at zero detuning of the generation frequency relative to the gain line centre ($\xi = 0$):

$$|E_1|^2 = |E_2|^2 = \frac{n - 1 - \delta_v(t)}{\alpha + \beta}, \qquad (6)$$

where $\alpha = \mathrm{Re}(\alpha_{1,2})$ and $\beta = \mathrm{Re}(\beta_{1,2})$. In accordance with (6), modulation of intracavity losses $\delta_v(t)$ causes a periodic change in the intensities of counterpropagating waves, which, in turn, leads to periodic modulation of the frequency dithering:

$$\dot{\Phi} = \dot{\varphi}_1 - \dot{\varphi}_2 = \delta_v(t)\frac{\alpha_{1i} - \alpha_{2i} + \beta_{1i} - \beta_{2i}}{\alpha + \beta}, \qquad (7)$$

where $\alpha_{1i,2i} = \mathrm{Im}(\alpha_{1,2})$ and $\beta_{1i,2i} = \mathrm{Im}(\beta_{1,2})$.

The change in the phase difference of counterpropagating waves during the dithering period T_d is determined by the integral

$$\Delta\Phi = \int_0^{T_d} \dot{\Phi}(t)\,\mathrm{d}t. \qquad (8)$$

Using formulae (5), (7), and (8) and assuming that the period of modulation of losses is equal to the dithering period, we obtain the expression for the vibrational zero shift $\Delta\Omega_v$:

$$\Delta\Omega_v = \Delta\Phi/T_d = \varepsilon\cos\varphi_v(\Delta\nu/\pi)\frac{\alpha_{1i} - \alpha_{2i} + \beta_{1i} - \beta_{2i}}{\alpha + \beta}. \qquad (9)$$

In accordance with (9), the zero shift $\Delta\Omega_v$ is proportional to the amplitude of the modulation loss εp_c. The maximum value of $\Delta\Omega_v$ occurs when the loss modulation is in-phase with the frequency dithering ($\varphi_v = 0$).

A vibrational zero shift also occurs when the loss is modulated at frequencies that are multiples of the dithering frequency $\omega_v = k/T_d$. For odd values of k, the zero shift $\Delta\Omega_v^k$ is determined by the formula

$$\Delta\Omega_v^k = \Delta\Omega_v/k, \qquad (10)$$

and for even k, a zero shift is absent ($\Delta\Omega_v^k = 0$).

2.2. Calculation results

Let us consider the results of calculations for a ZLG with a ring resonator perimeter length $L = 20$ cm, operating on one Ne isotope at a gas mixture pressure of 700 Pa. We assume that the intracavity losses per round trip for the light in the cavity are equal to 0.2%, and the modulation depth of losses due to mirror vibration is $\varepsilon = 0.1$. The switching frequency of the alternating dithering is $1/T_d = 250$ Hz. In papers [8, 9], formulae are given for calculating the coefficients that describe the polarisability of the gain medium. Using these formulae for the ZLG under study, we calculated the values of the parameters at the gain line centre: $\alpha = 2.3006$ and $\beta = 1.1674$; on the first half-period of the alternating frequency dithering, $\alpha_{1i} = \alpha_{2i} = -0.1257$ and $\beta_{1i} = \beta_{2i} = -0.0786$, and on the second, these coefficients have opposite signs. The amplitude of the alternating frequency dithering is $\Omega_d/2\pi = 48.2$ kHz.

Let us assume that the relative excess of the gain over losses is $n = 1.4$. In this case, in the absence of loss modulation, the relative intensities of the counterpropagating waves when tuned to the gain line centre are $\langle|E_1|^2\rangle = \langle|E_2|^2\rangle = 0.1125$. Consider the modulation of losses with a period equal to the dithering period ($k = 1$). Figure 1a shows the modulation of the intensity of the first wave, $|E_1|^2 - \langle|E_1|^2\rangle$, and Fig. 1b demonstrates the frequency dithering in the absence of modulation (dashed line) and in the presence of modulation (solid line). One can see from Fig. 1b that in-phase modulation of intracavity losses (φ_v) with an amplitude $\varepsilon p_c = 0.0002$ leads to a zero shift equal to of 1.72 kHz. For the ZLG K5 laser gyroscope, this shift (taking into account the scale factor) is 5000 deg h^{-1}.

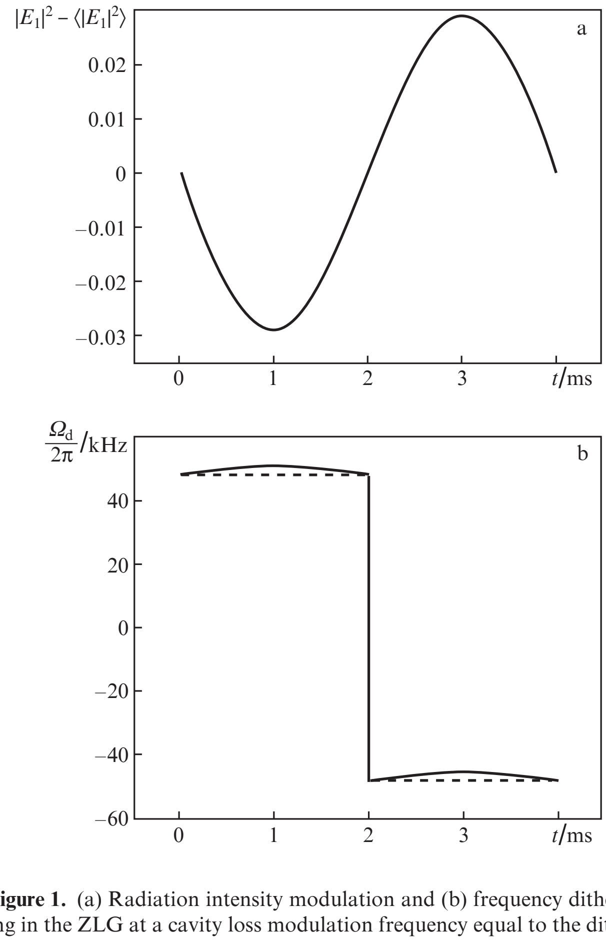

Figure 1. (a) Radiation intensity modulation and (b) frequency dithering in the ZLG at a cavity loss modulation frequency equal to the dithering frequency ($\omega_v = 2\pi/T_d$). The dashed line shows the frequency dithering in the absence of loss modulation.

Consider now the modulation of the loss with a frequency equal to the doubled dithering frequency (k = 2). Figure 2a shows the modulation of the intensity of the first wave, $|E_1|^2 - \langle|E_1|^2\rangle$, and Fig. 2b demonstares the shape of the frequency dithering in the absence (dashed line) and in the presence of modulation (solid line). As can be seen from Fig. 2b, the shape of the frequency dithering in this case is distorted due to the modulation of intracavity losses, but a zero shift is absent ($\Delta\Omega_{\rm v}^2 = 0$).

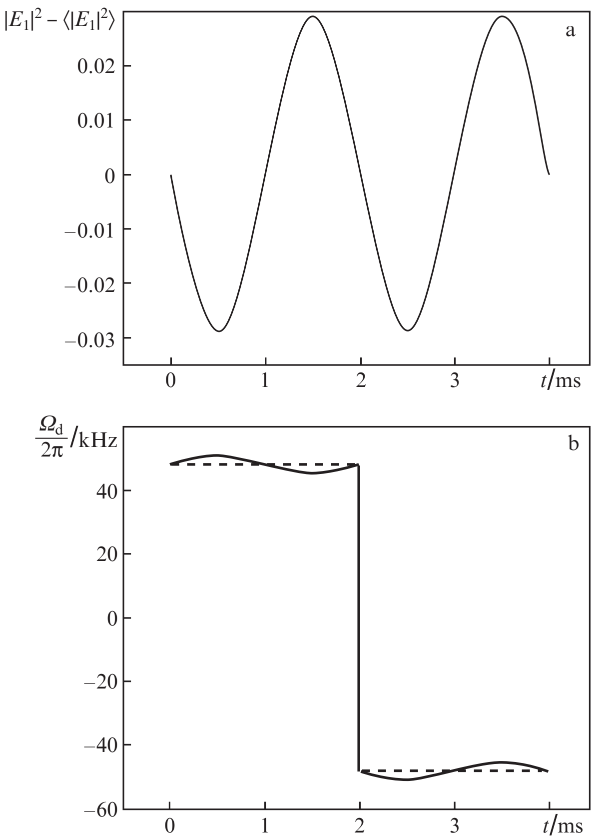

Figure 2. (a) Radiation intensity modulation and (b) frequency dithering in the ZLG at a cavity loss modulation frequency equal to the doubled dithering frequency ($\omega_{\rm v} = 4\pi/T_{\rm d}$). The dashed line shows the frequency dithering in the absence of loss modulation.

3. Results and discussion

The results of an experimental study of the vibrational zero shift are presented in Ref. [3]. The shift maxima were observed at odd harmonics of the dithering frequency ($\omega_{\rm v} = 2\pi k/T_{\rm d}$, k = 1, 3, …). The value of the first maximum (k = 1), equal to 250 deg h^{-1}, in accordance with the results given in Section 2, can be obtained for the amplitude of modulation of intracavity losses, $\varepsilon p_{\rm c} = 1 \times 10^{-5}$. Experimental studies [3] have shown that the zero shift linearly depends on the magnitude of vibration acceleration. This is consistent with the linear dependence $\Delta\Omega_{\rm v}^k$ on the amplitude of modulation of intracavity losses $\varepsilon p_{\rm c}$ in formulae (9) and (10).

For comparison, we present some results of the analysis of the vibrational zero shift during the modulation of the generation frequency detuning relative to the gain line centre ξ [3]. In this case, the vibrational shift of zero at odd harmonics of the dithering frequency turns out to be zero. To explain the experimentally observed maxima at these harmonics, it is assumed in [3] that a static oscillation frequency detuning occurs in the ZLG, which is associated with the imperfect operation of electronic units for adjusting the perimeter and pickups. The vibrational shift during frequency detuning modulation is proportional to the square of vibration acceleration, which is inconsistent with the experiment.

Thus, we have theoretically shown that the periodic modulation of intracavity losses at frequencies that are multiples of the frequency of the alternating dithering in the ZLG causes a periodic change in the shape of the frequency dithering. At odd harmonics, modulation leads to a zero shift of the ZLG. A mechanism is proposed for the vibrational shift of the ZLG zero due to modulation of intracavity losses during vibration of the resonator mirrors.

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