

Effect of elastic anisotropy on thermally induced distortions of a laser beam in single cubic syngony crystals with radial cooling. Part II

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Abstract. We report a study of thermally induced depolarisation of a laser beam in single cubic syngony crystals of 432, $\bar{4}3m$, and $m\bar{3}m$ symmetry groups with an anisotropic elastic stiffness tensor. For active elements in the form of a long rod and a thin disk with radial cooling under uniform volume pumping, we investigate the effect of anisotropy of elastic properties on the dependence of the degree of depolarisation in the crystal on the orientation of the crystallographic axes, as well as on the position of the critical orientation in the crystallographic coordinate system.

Keywords: high average power lasers, photoelastic effect, thermally induced depolarisation, anisotropy of cubic crystals.

1. Introduction

Thermal effects are an important factor limiting the power and quality of the output radiation of solid-state lasers [1, 2]. Thermally induced depolarisation of radiation in initially optically isotropic media – cubic single crystals, glasses, and ceramics – has been studied theoretically and experimentally since the 1960s. Except for few works, in cubic crystals only the anisotropy of the photoelastic effect is, as a rule, taken into account, whereas the elastic properties of materials are considered to be isotropic (see references in the first part of this paper [3]). However, already at the beginning of research it was known that this was only an approximation, and a solution to the elasticity problem for a parabolic temperature profile was found [4].

We study thermally induced beam distortions in single cubic syngony crystals of all symmetry groups taking into account the anisotropy of their elastic properties, as well as at an arbitrary orientation of the crystallographic axes. In [3], we obtained expressions for the phase and polarisation distortions of the beam in active elements in the form of a long rod and a thin disk. We found eigenpolarisations, as well as the arithmetic mean and the difference between the incursions of their phases; considered the specific orientations of crystals; and determined the effective values of the thermo-optical constants P and Q . This paper, the second part of the work, is devoted to the analysis of thermally induced depolarisation of radiation in elastically anisotropic single crystals of symmetry groups 432, $\bar{4}3m$, and $m\bar{3}m$ in comparison with the elastically

isotropic case and to finding optimal orientations of crystallographic axes under weak and strong birefringence in terms of minimisation of the degree of depolarisation. In Section 2, we introduce the notations necessary for the statement of the problem. In Section 3, thermally induced depolarisation in crystals of the simplest orientations is studied. In Section 4 we search for the optimal orientation in crystals with a positive parameter of photoelastic anisotropy, and in Section 5 we investigate the critical orientation in crystals with a negative value of this parameter. The results of the influence of the end effects on the degree of depolarisation in the rod, announced in [3], will be published elsewhere.

2. Statement of the problem and some notations

In the framework of this work, we restrict ourselves to media that do not rotate the polarisation plane. We will consider an active element in the form of a cylinder, the z axis of which coincides with the propagation direction of probe radiation. The set of methods for cutting an element from the crystal bulk is equivalent to the set of possible directions of the z axis in the crystallographic coordinate system (a , b , c), traditionally called crystal orientations. They are indexed using the first two Euler angles, i.e. azimuthal (α) and polar (β) (see Figs 1a, 1b, and 1d from [3]). The third Euler angle (Φ) is equal, with the opposite sign, to the angle of rotation of the active element around its axis (see Figs 1c and 1d from [3]).

Crystal orientations are also denoted by Miller indices $[MNP]$. In this paper, major attention is paid to $[M0N]$ and $[MMN]$ orientations shown in Fig. 1. Following the generally accepted approach, we will not distinguish between equivalent orientations, that is, those obtained from each other by cyclic permutation of indices ($[NPM]$, $[PMN]$) and changing the signs of indices ($[\overline{MNP}]$, $[\overline{M\bar{N}\bar{P}}]$, etc.) [5].

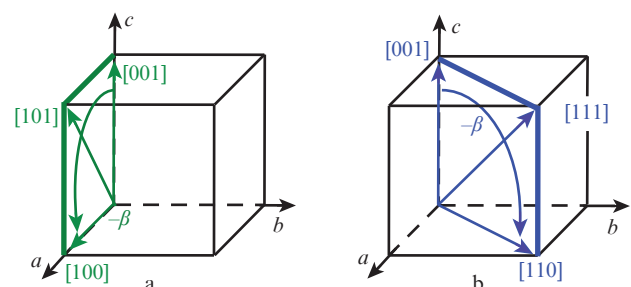


Figure 1. (a) Orientations $[M0N]$ ($\alpha = 0$) in the crystallographic coordinate system and (b) orientations $[MMN]$ ($\alpha = \pi/4$).

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Let us consider an active element in the form of a long rod or thin disk of radius R , heated uniformly throughout the volume and cooled through the lateral surface. The temperature and elastic stress fields are given in [3] and do not depend on z , if we neglect the end effects in the rod.

The lateral cooling of a thin disk is not optimal, but it allows the degree of depolarisation to be calculated analytically. We believe that the regularities we have identified will be applicable in the case of end-face heat removal, despite the impossibility of a quantitative assessment of thermally induced beam distortions.

The local degree of depolarisation Γ is commonly referred to as the fraction of the radiation intensity that has passed during propagation through a birefringent optical system into the polarisation that is orthogonal to the initial one. A similar fraction of power is called the integral degree of depolarisation. In the plane wave approximation, it is easy to obtain that after a laser beam, initially linearly polarised in the perpendicular plane xy at an angle θ to the x axis, passes along the axis of one active element, the permittivity tensor of which is independent of z , the local degree of depolarisation is expressed as

$$\Gamma(x, y) = \sin^2\left(\frac{\delta}{2}\right) \sin^2 2(\Psi - \theta), \quad (1)$$

where $\Psi(x, y)$ is the angle of inclination of the polarisation vector of one of the eigenwaves in the active element; and $\delta(x, y)$ is the difference between the incursions of their phases. Averaging (1) over the cross section of the probe beam with the field intensity $\mathbf{E}_{\text{in}}(x, y)$, we obtain the integral degree of depolarisation

$$\gamma = \left(\iint_S |\mathbf{E}_{\text{in}}|^2 dS \right)^{-1} \iint_S \Gamma |\mathbf{E}_{\text{in}}|^2 dS \quad (2)$$

(for simplicity, we assume that there is no absorption and amplification of probe radiation in the medium). With strong birefringence, γ tends to a steady-state value [6]

$$\gamma_\infty = \left(\iint_S |\mathbf{E}_{\text{in}}|^2 dS \right)^{-1} \iint_S \tilde{\Gamma}_\infty |\mathbf{E}_{\text{in}}|^2 dS, \quad (3)$$

where

$$\tilde{\Gamma}_\infty = \frac{1}{2} \sin^2 2(\Psi - \theta).$$

In [3] we obtained expressions for δ and Ψ in crystals with anisotropic elastic properties. Without going into details here, we note that they differ in a thin disk and a long rod, as well as that δ is proportional to the dimensionless heat release power

$$p = \frac{QP_\Sigma}{\lambda\kappa}, \quad (4)$$

and Ψ does not depend on it. Here P_Σ is the total power of heat release in the active element; λ is the wavelength of probe radiation in free space; κ is the thermal conductivity coefficient; and Q is the thermo-optical constant, which differs in the disk and the rod and, with this reservation, is a material parameter {see (66) in [3]}:

$$Q_{\text{disk}} \propto E_{[001]}, \quad (5)$$

$$Q_{\text{rod}} = \frac{Q_{\text{disk}}}{1 - \nu_{[001]}}.$$

Here, in accordance with [7]

$$E_{[001]} = 1/s_{11}, \quad (6)$$

$$\nu_{[001]} = -s_{12}/s_{11},$$

are Young's modulus and Poisson's ratio for the [001] orientation, respectively; and \mathbf{s} is the elastic compliance tensor.

Cubic crystals can be divided into two types according to the form of their material tensors. In this work, we study crystals of the $m\bar{3}m$, 432 , and $\bar{4}3m$ symmetry groups, which in [3] we agreed to call $m\bar{3}m$ crystals. Their elastic compliance tensor \mathbf{s} and piezo-optical tensor $\boldsymbol{\pi}$ have the same form and are determined by three independent nonzero coefficients {see [8] and Fig. 2a from [3]}.

The dependence of the thermally induced beam distortions on the orientation of the crystallographic axes in $m\bar{3}m$ crystals in the approximation of the isotropic elasticity problem is determined by the photoelastic anisotropy parameter ξ [6, 9], for which in [3] we chose the piezo-optical ratio, which in the Nye notation has the form

$$\xi_\pi = \pi_{66}/(\pi_{11} - \pi_{12}). \quad (7)$$

We also showed that in media with anisotropic elastic properties, this dependence is modulated by the functions that are determined by the combination of $\nu_{[001]}$ with the elastic anisotropy parameter

$$\xi_s = s_{66}/[2(s_{11} - s_{12})]. \quad (8)$$

Note that in elastically anisotropic media expression (7) differs from the elasto-optical ratio {see (59a) in [3]} used in [6, 9] and other works by a factor of ξ_s .

For different orientations of the crystallographic axes, let us compare the degree of thermally induced depolarisation calculated for elastically anisotropic cubic single crystals in the form of a long rod and a thin disk with that calculated in the isotropic elasticity approximation. Several special cases will be considered in which the dependences of the degree of depolarisation on the crystal orientation are qualitatively different. We will focus on choosing the optimal orientation. Since in the experiment the angle of rotation Φ , in contrast to the other Euler angles, can be easily changed after the manufacturing of the active element, we will investigate the degree of depolarisation that is minimal with respect to Φ , and will call it optimal. To simplify the calculation we will assume that the probe beam has a flat-top transverse profile of the radiation intensity. Such beams are often generated in high-power multistage laser systems. Moreover, this assumption does not qualitatively affect the results. The beam radius is denoted by r_0 .

3. Thermal depolarisation in active elements made of $m\bar{3}m$ single crystals of the simplest orientations

Figure 2 shows the optimal integral degree of depolarisation in single crystals with three simplest orientations ([001], [011],

and [111]) as a function of the dimensionless heat release power p (4). The calculations were performed for CaF_2 , SrF_2 , KCl and medium 1, i.e. a model crystal with photoelastic properties as in YAG and elastic properties as in CaF_2 . The material parameters of these and other media used in the calculations are given in Table 1.

Table 1. Material properties of real and model m3m crystals used in the calculations.

Medium	ξ_π	ξ_s	$\nu_{[001]}$
CaF_2	-0.47 [11, 12]	1.77 [13]	0.212 [13]
SrF_2	-0.284 [12]	1.25 [14]	0.266 [14]
KCl	-2.32 [15]	2.69 [14]	0.135 [14]
YAG	3.2 [2]	0.965 [14]	0.25 [14]
LiF	0.49 [16]	0.53 [17]	0.287 [17]
1	3.2	1.77	0.212
2	3.2	0.44	0.212
3	0.49	2.69	0.135

One can see from Fig. 2 that the dependences for elastically anisotropic media are qualitatively the same as for elastically isotropic media: quadratic growth at low powers and damped oscillations around the steady-state value of γ_∞ (3) at high powers [10]. Note that a comparison of the elastic anisotropic and elastic isotropic dependences at the same p means

that the latter are plotted using the values of the elastic isotropic Young's modulus and Poisson's ratio determined for the [001] orientation (6):

$$E = E_{[001]}, \quad (9)$$

$$\nu = \nu_{[001]},$$

which may not correspond to the tabular data presented in the literature. In the case of discrepancy, the dependences of the degree of depolarisation on the dimensional power P_Σ can relate to each other differently, in particular, be much closer to each other.

From the expressions given in [3], it is easy to obtain that in a thin disk the phase difference δ , proportional to the dimensionless heat release power p , differs from the elastically isotropic one only by the additional factor $1/Z_{\text{disk}}$, which depends on the orientation of the crystal axes, while the angle of inclination of the polarisation vector of the eigenwave Ψ , in the expression for which Z_{disk} cancels out together with p , is equal to this angle in the isotropic elasticity approximation {see (32), (63) in [3]}. Therefore, these pairs of dependences are obtained from each other by compression or stretching along the horizontal axis, while the values of γ_∞ are the same. Recall that the value of Z is the ratio of in-plane stresses in the elastically isotropic approximation to the stresses in the elastically anisotropic calculation:

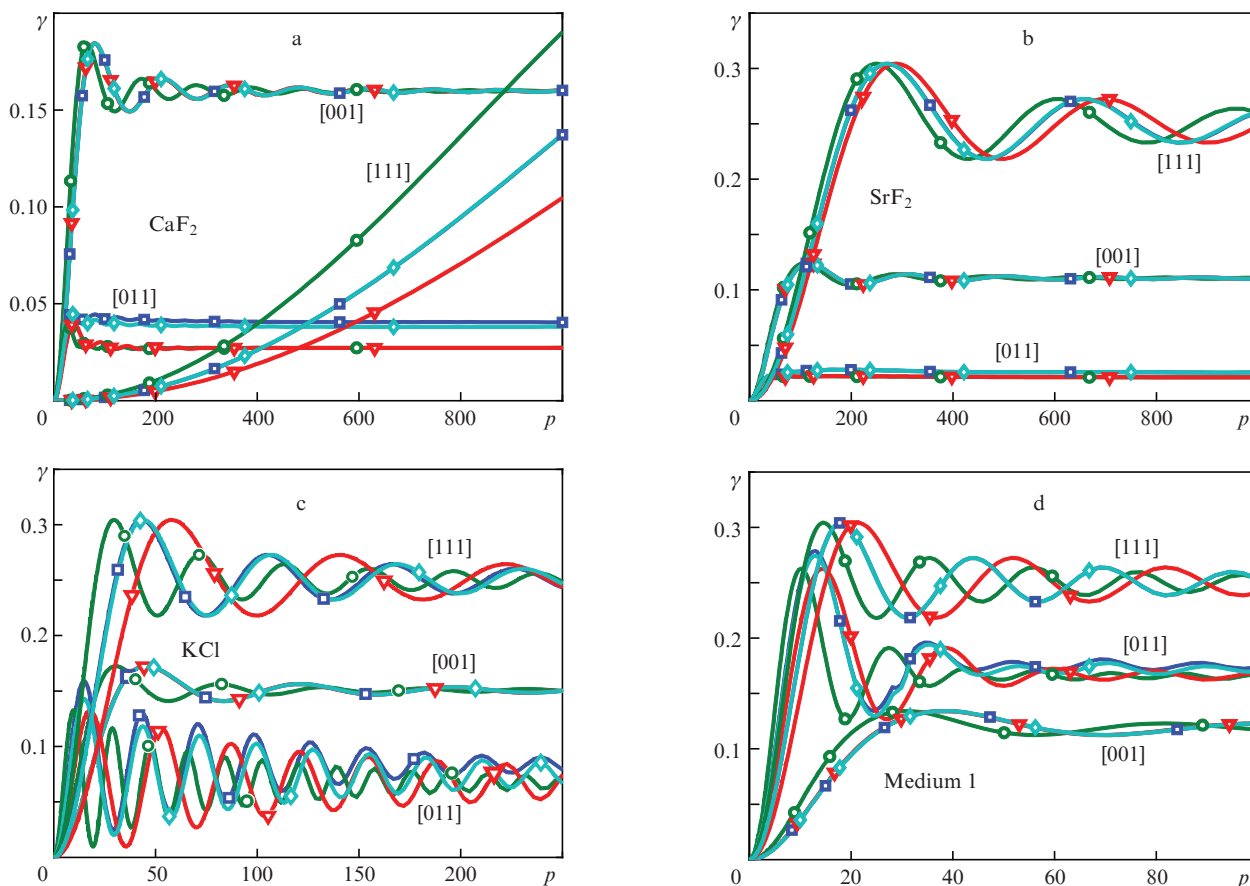


Figure 2. (Colour online) Calculated analytically optimal integral degrees of depolarisation as functions of the dimensionless power of heat release in elements made of various crystals with three simplest orientations of the axes at $r_0 = 0.5R$. The calculations were performed for elastic stresses calculated in the elastically isotropic approximation (green circles), for a crystal in the form of a thin disk (red triangles), as well as for the complete (dark blue squares) and simplified (blue diamonds) Sirotin solutions for a long rod.

$$Z = \sigma_{\perp}^{\text{iso}}/\sigma_{\perp}. \quad (10)$$

With the material parameters used in the calculation, $Z_{\text{disk}} > 1$; therefore, the degree of depolarisation in the disk grows and reaches the level γ_{∞} more slowly than in the isotropic elasticity approximation.

For crystals in the form of a long rod, we use Sirotin's solution [4], in which the factor Z_s also differs from unity, but weakly depends on the crystal orientation: it is seen from Fig. 3a that near $\xi_s = 1$ the ratio of in-plane stresses

$$\frac{\sigma_{\perp}([MNP])}{\sigma_{\perp}([001])} = \frac{Z([001])}{Z([MNP])} \quad (11)$$

depends linearly and quadratically on the elastic anisotropy parameter in a disk and a rod made of crystals with the simplest orientations of the crystallographic axes, respectively. We also proposed a simplified solution {see (51) in [3]}, in which this dependence was neglected. The second significant difference of the solution for a long rod from the elastically isotropic approximation is the inequality of the stigmatic

component of the stress σ_{zz} to the sum $\sigma_{rr} + \sigma_{\varphi\varphi}$. This difference is expressed using the coefficients D_{01} in the simplified solution and D_1 in the complete one {(45) taking into account (31) from [3]}:

$$D_{01} \approx \frac{\int_0^{2\pi} \sigma_{zz} d\varphi}{2\pi(\sigma_{rr} + \sigma_{\varphi\varphi})} = Z_s \frac{\int_0^{2\pi} \sigma_{zz} d\varphi}{2\pi\sigma_{zz}^{\text{iso}}}. \quad (12)$$

Sirotin's solution also differs from the elastically isotropic one by the presence of azimuthal variation of σ_{zz} and nonzero stresses σ_{xz} and σ_{yz} . In the simplified solution, and for crystals with the simplest orientations of the axes in the complete one, $\sigma_{xz} = \sigma_{yz} = 0$.

The difference of the coefficients D_{01} and D_1 from unity, depending on ξ_s , can be greatest for the [001] or [111] orientation (see Fig. 3b and Fig. 3 in [3]). However, in $m3m$ crystals with the indicated orientations, these quantities do not affect birefringence due to the fact that the coefficients $A_{1\phi}$ and $B_{1\phi}$ are equal to zero in the expressions for δ and Ψ {see (65), (67) in [3]}. In this regard, the effect is observed in Fig. 2 only for the [011] orientation and is as follows. Analysis of the stress field in the rod {(51) and (A2.6) in [3]} shows that at $\xi_s > 1$ and $v_{[001]} > 0$ the values of D_{01} and D_1 for crystals with this orientation are less than unity (Fig. 3b), which causes an increase in the coefficient I_{∞} {see (91) in [3]}. This means an increase in the misorientation of the polarisation vectors of eigenwaves in the paraxial region of the active element, which causes an increase in γ_{∞} at a small radius of the probe beam (see [10] and section 5.2.4 in [3]). In accordance with this fact, in all dependences in Fig. 2, the degree of depolarisation with strong birefringence in a rod with the [011] orientation is greater than the elastically isotropic estimate. In addition, the degrees of depolarisation for the simplified and complete solutions for a long rod are, as a rule, very close (in Section 5, the critical orientation, near which the differences are significant, will be considered).

Note that for the considered fluoride and KCl crystals with weak birefringence, the [111] orientation turns out to be better than the [001] orientation (see also [11, 18]), while in many media, for example, in YAG and TGG, the situation is opposite [5, 6, 19]. It was shown in [11] that the [111] orientation is preferable to the [001] orientation for media with $-2 < \xi_{\pi} < -0.2$, and for CaF_2 , whose ξ_{π} parameter is close to -0.5 , the advantage of the [111] orientation is especially pronounced, since at this value it coincides with the critical orientation.

We also note that the dependences in Fig. 2 are plotted up to very high thermal powers, which are not achievable in practice due to mechanical destruction of materials, and their right-hand sides are only of academic interest. Thus, for a 10-cm-long CaF_2 rod, the ultimate strength corresponds to $p_{\text{max}} = 90$ at a wavelength of $1 \mu\text{m}$, and for KCl, under the same conditions, $p_{\text{max}} = 15$ [20]. In addition, diffraction effects are not taken into account in the plane wave approximation.

From the point of view of choosing the optimal orientation, $m3m$ crystals can be divided into two classes. In media with $\xi_{\pi} < 0$, there is a critical orientation of the $[MMN]$ type (see Section 5.2.1 in [3]), in which the axes of the permittivity tensor in the entire cross section of the active element are aligned in one direction, and therefore depolarisation can theoretically vanish [21, 22]. This orientation is preferable and, therefore, comparing the dependences in Figs 2a–2c with each other does not make much practical sense. The

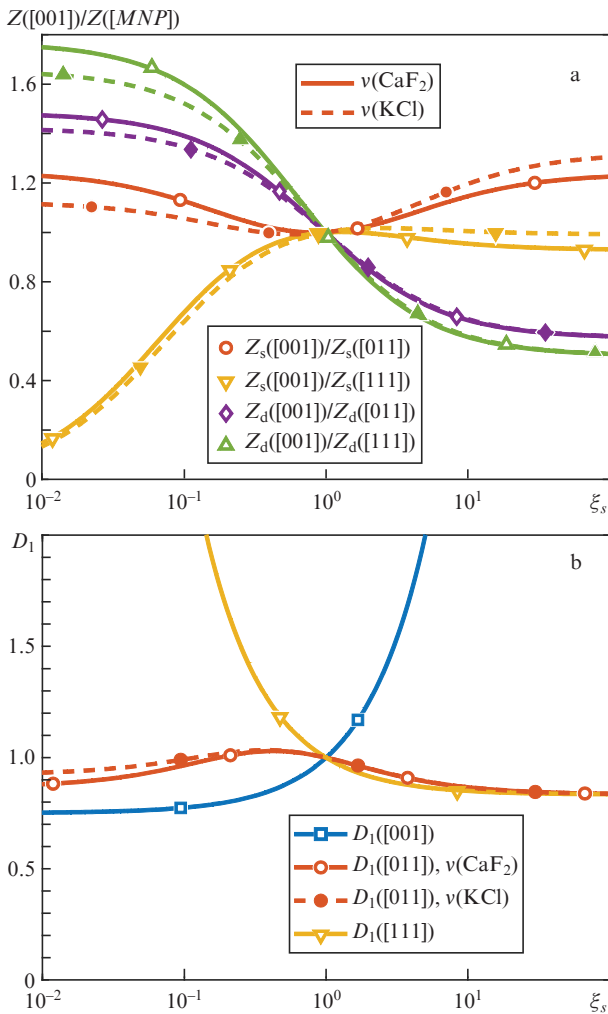


Figure 3. (Colour online) Elastic anisotropy parameter for media with the coefficient $v_{[001]}$, equal to $v_{[001]}$ for CaF_2 and KCl vs. (a) variations in in-plane stresses in disks and rods made of crystals with the simplest orientations of crystallographic axes and (b) normalised stigmatic components of σ_{zz} (12) in the rod.

position of the critical orientation in elastically anisotropic media will be considered in Section 5. In $m3m$ crystals with $\xi_\pi > 0$, the optimal orientation in the elastically isotropic approximation is, as a rule, either $[001]$ or $[011]$ orientation $[6, 10]$. This case will be considered in Section 4.

4. Thermal depolarisation in $m3m$ single crystals in the absence of critical orientation

The dependence of the degree of depolarisation on orientation is qualitatively determined not only by the sign of the

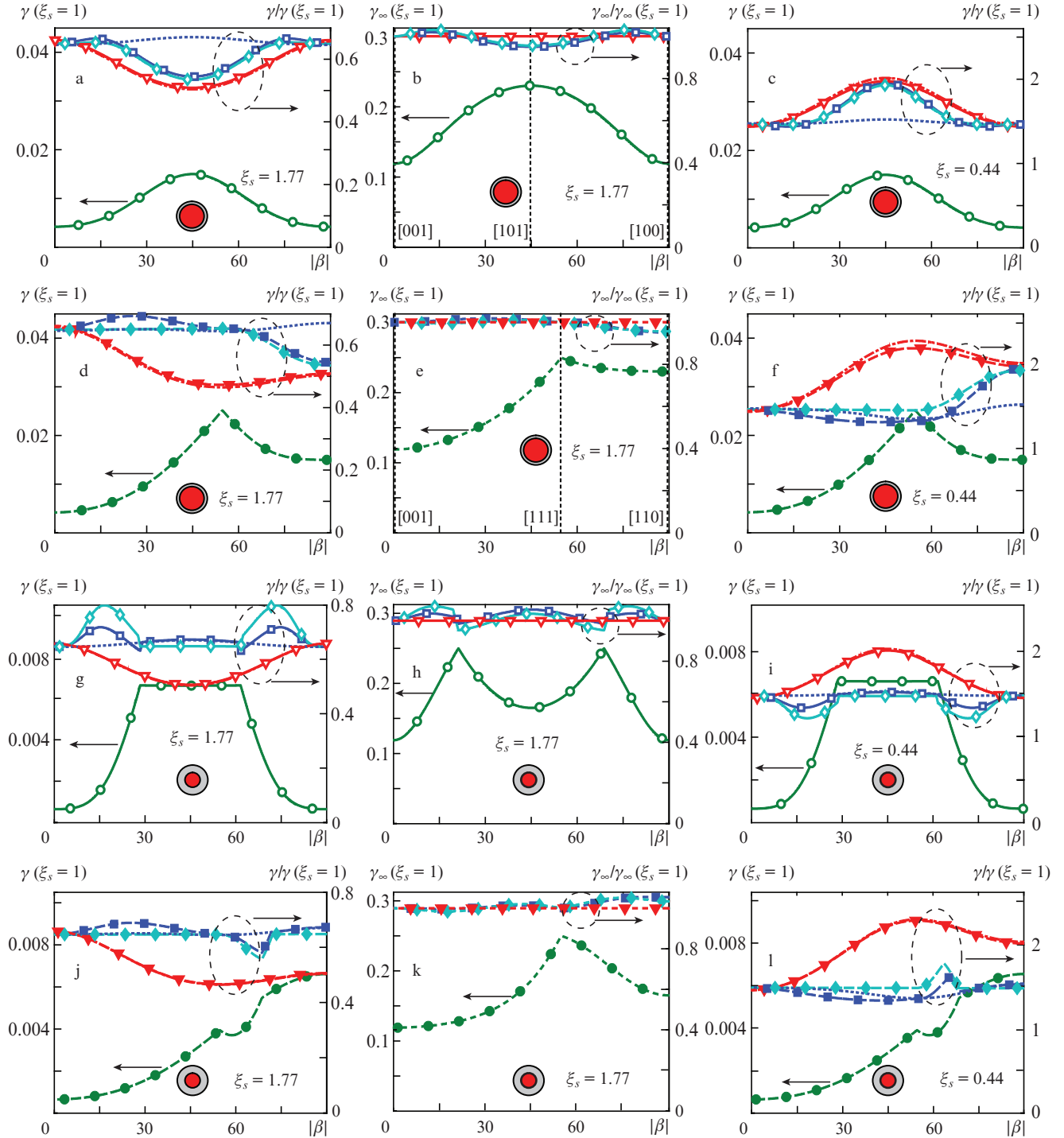


Figure 4. (Colour online) Calculated analytically optimal integral degrees of depolarisation in the elastically isotropic approximation (green circles, left axes) and the ratio of the optimal integral degrees of depolarisation (right axes) to them in the approximation of a thin disk (red triangles), a long rod [the stress field is calculated by the complete (dark blue squares) and simplified (blue diamonds) Sirotnin solutions] made of model media 1 (a, b, d, e, g, h, j, k) and 2 (c, f, i, l) as functions of the Euler angle β . The calculation was performed at dimensionless heat release powers $p = 1$ (a, c, d, f, g, i, j, l) and $p \gg 1$ (b, e, h, k) for orientations $[MON]$ ($\alpha = 0$, a–c, g–i, solid curves) and $[MMN]$ ($\alpha = \pi/4$, d–f, j–l, dashed curves). The probe beam radius is $0.8R$ (a–f) and $0.5R$ (g–l) and is schematically shown in the form of circles. For $p = 1$, $Z_s^{-2}(\beta)$ (blue dotted curves) and $Z_{\text{disk}}^{-2}(\beta)$ (red dash-dotted curves) are shown. The simplest orientations are indicated by vertical dashed lines (b, e).

parameter ξ_π , but also by whether its modulus is greater or less than unity.

4.1. Case of $\xi_\pi > 1$

The ratios of the optimal degrees of depolarisation in anisotropic and elastically isotropic calculations for model media with the most interesting orientations, such as $[M0M]$ and $[MMN]$ (see Fig. 1), are shown in Fig. 4. With relatively low birefringence ($p = 1$, Figs 4a, 4d, 4g, 4j; 4c, 4f, 4i, 4l), this ratio for a thin disk is close to Z_{disk}^{-2} , the value that it is equal to in the weak birefringence approximation {see (54), (63) in [3]}. For $p \gg 1$ (Figs 4b, 4e, 4h, 4k), the ratio for the disk is always equal to unity, since γ_∞ does not depend on δ and, therefore, on Z_{disk} .

For the rod, the ratio of the degrees of depolarisation deviates from Z_s^{-2} . This difference, as noted in Section 3, is mainly due to the behaviour of the D_1 (or D_{01}) coefficient.

Deviations of the degree of depolarisation from the results of the elastic-isotropic calculation are more pronounced under weak birefringence than under strong one. With weak birefringence and under condition (9), they can be, depending on the values of the material parameters, twice greater or twice less. In this case, the variation in the ratios of the optimal degrees of depolarisation, depending on the orientation of the crystal, can reach tens of percent (twice as much as in Fig. 3a); therefore, taking into account the anisotropy of elastic properties is important for correct calculation. With strong birefringence, both deviations are 5%–10%, and their dependence on orientation is more complex.

On the whole, in the considered case, the effect on the degree of depolarisation of the anisotropy of the elastic properties of the medium is weaker than the anisotropy of the photoelastic properties; therefore, the optimal orientations of the axes in the elastically anisotropic case are generally the same as in the elastic-isotropic case. In Fig. 4 and Fig. 2d, the optimal orientation for any heat release power is the $[001]$ orientation; however, with a decrease in the probe beam radius under strong birefringence, the $[011]$ orientation becomes preferable. The disadvantage of this regime is the ineffective use of the pump radiation and the volume of the crystal. Mukhin et al. [10] showed that with a more practically favourable synchronous decrease in the radii of the pump and probe radiation beams, the $[011]$ orientation will be the best with an even less efficient use of the crystal volume. This regime cannot be investigated analytically in media with anisotropic elasticity, since the expressions for the stress field are unknown to us.

Theoretically, a situation is possible when elastic anisotropy is comparable to or stronger than photoelastic, and the degree of depolarisation in an elastically anisotropic medium is qualitatively different from elastically isotropic. In this case, the optimal orientations may also change. This rather specific case will be considered in the third part of the work.

4.2. Case of $0 < \xi_\pi < 1$

When $|\xi_\pi| < 1$, the optimal degree of depolarisation in the elastically isotropic approximation for all orientations of the form $[M0M]$ in the weak birefringence approximation is the same, and for $\xi_\pi > 0$ it is also minimal among all orientations. In elastically anisotropic media, this constancy is not observed, which is illustrated in Figs 5a and 5b. In a thin disk, the deviation from the results of the elastic isotropic calculation is due only to the value of Z_{disk} . As follows from Fig. 3a,

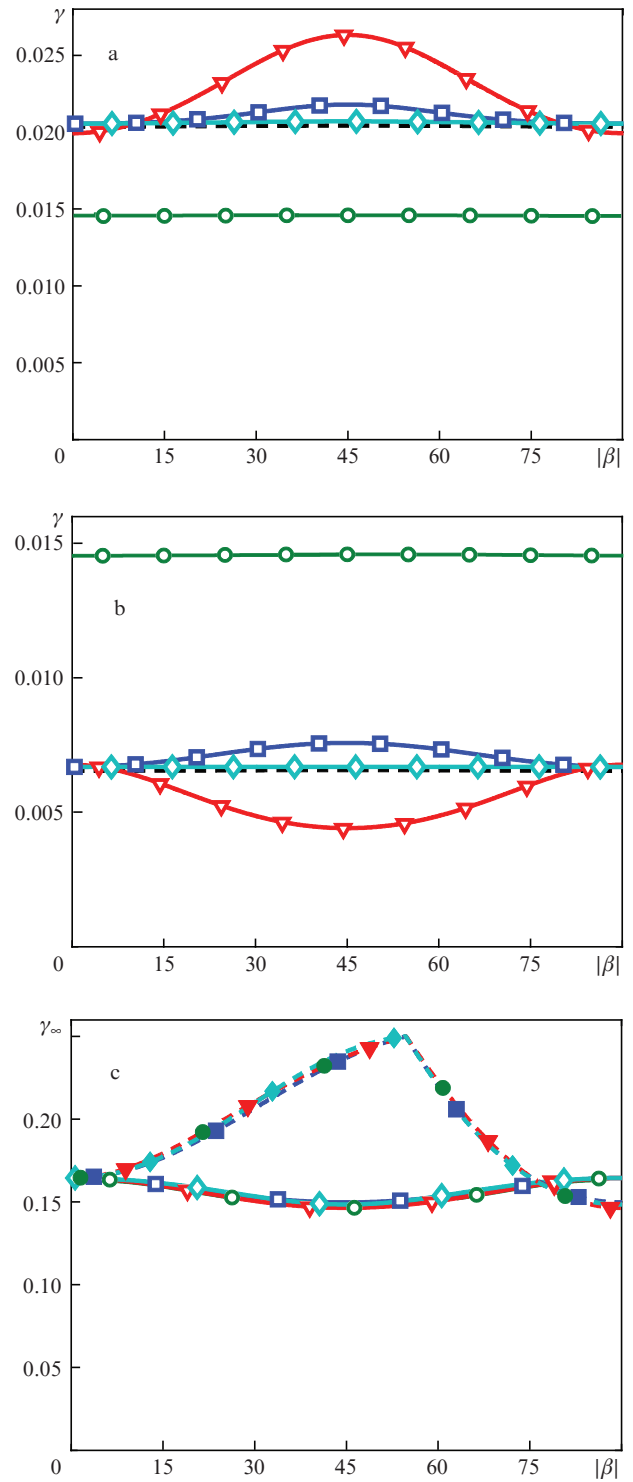


Figure 5. (Colour online) Calculated analytically optimal integral degrees of depolarisation in LiF (a, c) and in medium 3 (b) for the orientations $[M0N]$ ($\alpha = 0$, solid curves) and $[MMN]$ ($\alpha = \pi/4$, dashed curves) at $p = 5$ (a, b) and $p \gg 1$ (c) in a disk (red triangles), in a long rod [stresses are calculated by the complete (dark blue squares) and simplified (blue diamonds) Sirotn solutions], in the elastically isotropic approximation (green circles, and at $p = 5$ also a scaled graph, dashed line). The probe beam radius is $0.7R$.

for $\xi_s < 1$ the optimal orientation is $[001]$, and for $\xi_s > 1$ the optimal orientation is $[011]$. In a simplified solution for a rod, the degree of depolarisation in crystals with the $[M0N]$ orientations is almost constant. This indicates that its behaviour in

the exact solution is also mainly due to the magnitude of the in-plane stresses. In most cases, with weak birefringence, the optimal orientation in the rod will be [001], since the value of $1/Z_s$ for this orientation is less than for [011].

With strong birefringence, the [011] orientation is optimal even at large probe beam radii (Fig. 5c). Since, as noted above, the difference between the stress σ_{zz} and the elastically isotropic stress in the rod for this orientation is small, the elastic anisotropy is weakly manifested.

Thus, for $\xi_\pi > 1$, as well as for $0 < \xi_\pi < 1$ and strong birefringence, the optimal orientations in an elastically anisotropic crystal, as a rule, coincide with those found earlier in the elastically isotropic approximation. At $0 < \xi_\pi < 1$ and weak birefringence, they are entirely determined by the elastic properties of the medium.

5. Critical orientation in m3m single crystals with a negative parameter of photoelastic anisotropy

Figure 6 shows the dependences of the optimal degree of depolarisation on the Euler angle β under relatively weak birefringence in CaF_2 , SrF_2 , and KCl with the $[MMN]$ orientations. For comparison, we also present the dependences for a long rod, obtained using Parfenov's solution to the elasticity problem [23]. Calculations show that for a thin disk the shape of the dependence differs from the approximation of isotropic elasticity; however, when Sirotin's solution for a long rod is used to calculate the stress field, the differences are small and are most pronounced in the vicinity of the [110] orientation (as our calculations show, this difference is most significant at $|\xi_\pi| > 1.5\text{--}2.0$ and a small diameter of the probe beam).

It can be seen from Fig. 6 that the simplest orientation [111] is a point of a local maximum of the optimal degree of depolarisation γ , even when for this orientation it is less than for [001]. The degree of depolarisation decreases to zero for the critical orientation, and in a thin disk and when the simplified Sirotin solution for a long rod is used to calculate stress fields, it coincides with the critical orientation in the elastically isotropic approximation [[C]], which is determined by the Euler angles

$$\begin{aligned} \alpha &= \pi/4 + \pi k/2, \\ \tan^2 \beta &= -\xi^{-1}, \end{aligned} \quad (13)$$

where $\xi = \xi_\pi$ (see Section 5.2.1 in [3]). When use is made of the complete Sirotin solution for a long rod, the critical orientation $[[C_s]]$ can deviate from [[C]] by a value from fractions to units of a degree. The reason for the discrepancy is the inequality of the stresses σ_{xz} and σ_{yz} to zero. Parfenov's solution to the elasticity problem in a cubic crystal is a generalised plane deformation (see Section 4.3.2 in [3]); therefore, the critical orientation in it is determined by expression (13) with $\xi = \xi_p$ {see (59a) in [3]} [21] and, as can be seen from Fig. 6 and as shown theoretically in Section 5.2.1 of [3], is determined with a large error.

Let us consider in more detail the behaviour of the degree of depolarisation in crystals with a close-to-critical orientation of the axes. Figure 7 shows the numerically calculated angles $\delta_{[[C]]}$ of deviation of the $[[C_s]]$ direction in a long rod

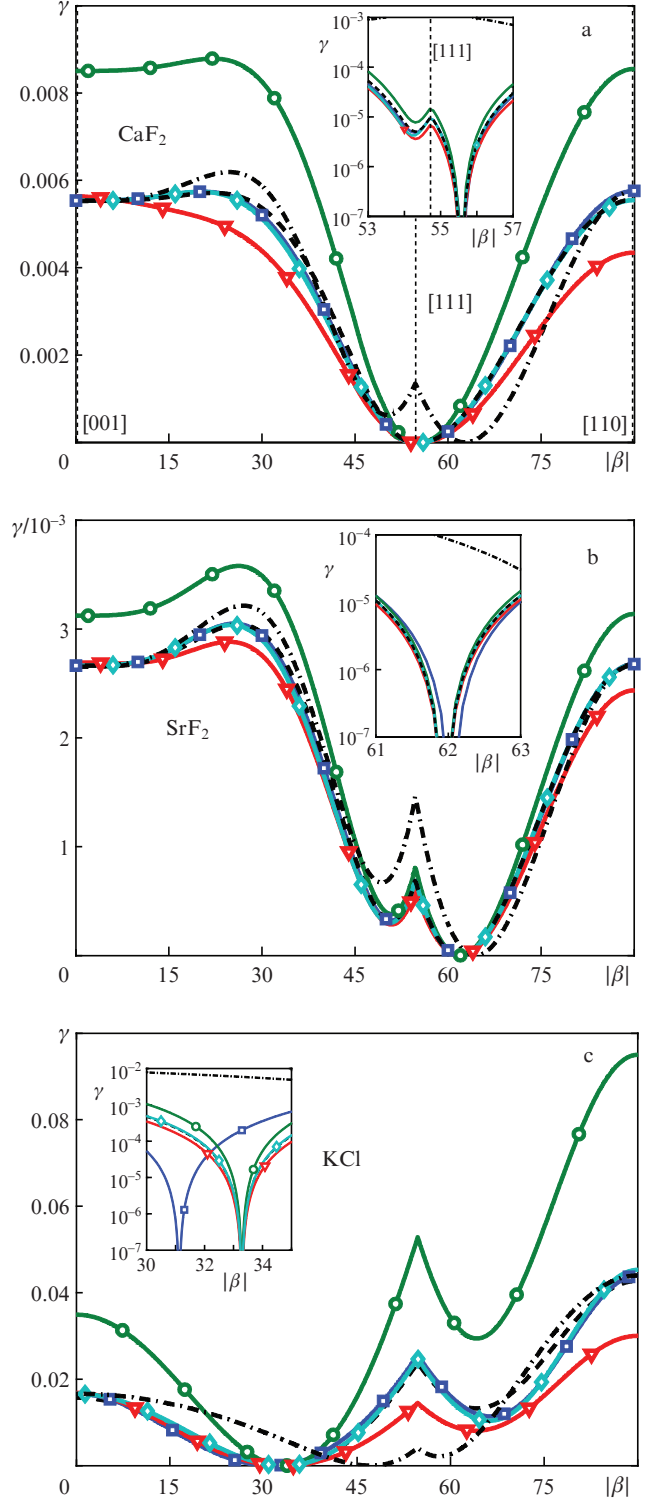


Figure 6. (Colour online) Calculated analytically optimal integral degrees of depolarisation as a function of the Euler angle β in (a) CaF_2 , (b) SrF_2 , and (c) KCl with orientations $[MMN]$ ($\alpha = \pi/4$) at $p = 3$. The probe beam radius is $0.8R$. The calculations were performed in the approximation of isotropic elasticity (green circles, as well as the scaled dependences – black dashed curves) for crystals in the form of a thin disk (red triangles), for crystals in the form of a long rod [the stress field is calculated by the complete (dark blue squares) and simplified (blue diamonds) Sirotin solutions], as well as for crystals in the form of a long rod using Parfenov's solution to the elasticity problem – black dash-dotted curves. The simplest orientations are indicated by vertical dashed lines (a).

from the estimated value $[[C]]$, as well as the relative values of the residual depolarisation

$$\gamma_N = \frac{\gamma([[C]])}{\min[\gamma([001]), \gamma([011]), \gamma([111])]}, \quad (14)$$

that is, optimal integral degrees of depolarisation normalised by the minimum among the three simplest orientations ([001], [011], and [111]), corresponding to this estimate in CaF_2 , KCl , and SrF_2 . Figure 7 also presents the dependence of angles of deviation and residual depolarisation on ξ_π in model media with parameters $\nu_{[001]}$ and ξ_s equal to these parameters for CaF_2 and for KCl . One can see that at $-1 < \xi_\pi < 0$, the differences in the positions of the two orientations do not exceed 1° and vanish at $\xi_\pi = -0.5$. These differences, as a rule, are insignificant from a practical point of view, since they correspond to the lateral drift of the beam by 1–2 mm over a crystal length of 10 cm and, in experimental implementation, can be compensated for by adjusting the optical system. Accurate observance of the critical orientation at the stage of sample preparation can be essential in lasers with a multi-kilowatt average power level and diffraction quality of the beam, as well as in cases where alignment of the system is difficult.

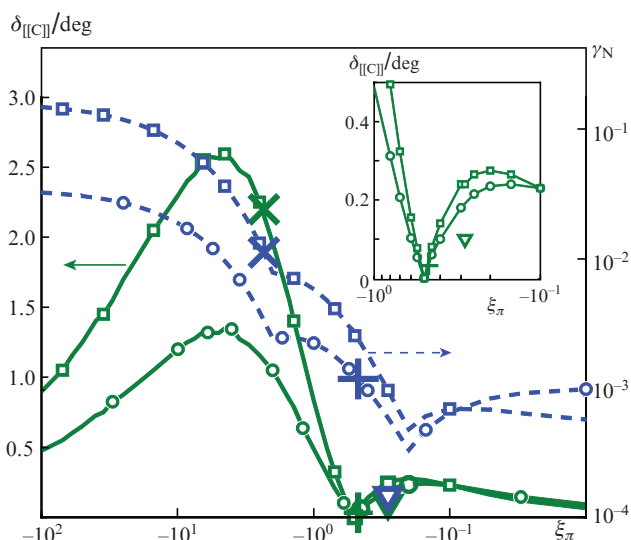


Figure 7. (Colour online) Angles $\delta_{[[C]]}$ of deviations of the direction $[[C]]$ from the estimated value $[[C]]$ (solid curves) and normalised optimal integral degrees of depolarisation γ_N for the orientation $[[C]]$ (dashed curves) at $p = 3$ and $r_0 = 0.8R$ in CaF_2 (+), SrF_2 (∇) and KCl (\times), and as functions of ξ_π in media with parameters $\nu_{[001]}$ and ξ_s equal to these parameters for CaF_2 (circles) and for KCl (squares).

At $\xi_\pi < -2$, the angle $\delta_{[[C]]}$ with strong elastic anisotropy increases to several degrees. With a further decrease in ξ_π , the critical orientation tends to [001] and the angle tends to zero, but γ_N in this case grows to ~ 0.1 , which makes a loss in the degree of depolarisation from missing the critical orientation more significant.

It can be seen from Figs 6 and 7 that for crystals with a close-to-critical orientation, the relative changes in the calculated degree of depolarisation in the rod due to its smallness in absolute value turn out to be very sensitive to the use of various approximations. However, in the case of using the analytical estimate $[[C]]$ to determine the critical orientation $[[C_s]]$ in a long rod, the residual degree of depolarisation still

remains two to three orders of magnitude less than for the best of the simplest orientations; for $|\xi_\pi|$, it is much less than or of the order of unity and an order of magnitude less for $|\xi_\pi| \gg 1$. Nevertheless, it should be borne in mind that, due to the sharp dependence of the degree of depolarisation on orientation, the difference between a real experiment and an idealised computational model can lead to strong deviations of experimental results from theoretical predictions. The reasons may be, in particular, nonobservance of the geometry of the long rod, which makes the influence of its end faces noticeable, as well as the presence of defects in the crystal structure of the sample, which cause constant built-in elastic stresses and, as a consequence, the so-called cold distortions of the beam. In particular, in an experiment with CaF_2 [11], it was possible to achieve a 20 times lower degree of depolarisation than for the [001] orientation; however, the calculated degree of depolarisation for the [111] orientation – the local maximum in the inset in Fig. 6a – should have been still approximately 25 times less. It was noted in [11] that such a result could be obtained, in particular, when the crystal orientation deviates by 2° from the optimal one, which also follows from the inset in Fig. 6a.

6. Conclusions

We have studied the thermally induced depolarisation of a laser beam in cylindrical active elements in the form of a long rod and a thin disk made of single cubic syngony crystals of symmetry groups 432, $\bar{4}3m$, and $m\bar{3}m$ with an anisotropic elastic stiffness tensor under volume uniform pumping and lateral heat removal. The dependence of the degree of thermally induced depolarisation on the orientation of the crystallographic axes of crystals is analysed for different values of the photoelastic anisotropy parameter ξ_π . With weak birefringence, the difference between the degree of depolarisation and the elastically isotropic approximation can reach, depending on the method for determining the isotropic elastic moduli and on the degree of anisotropy of elastic properties, from tens of percent to two times. The shapes of the dependences of the degree of depolarisation on the orientation of the axes also differ, with differences being manifested in a long rod and in a thin disk. Discrepancy in the disk is generally more pronounced.

There are three ranges of values of the photoelastic anisotropy parameter, in which elastic anisotropy affects differently the choice of the optimal orientation. For $\xi_\pi > 1$, the optimal orientations in elastically anisotropic and elastically isotropic calculations, as a rule, coincide. At $0 < \xi_\pi < 1$ and with weak birefringence, the optimal orientation is determined only by the elastic properties of the medium and does not depend on the photoelastic properties, and with strong birefringence it generally coincides with the optimal elastically isotropic orientation.

For crystals with $\xi_\pi < 0$, there is a critical orientation at which thermally induced depolarisation vanishes in theory. The position of this orientation in the disk is identical to that obtained in the elastically isotropic calculation, and in the long rod it differs by an angle of the order of a degree, which is much less than follows from the previous estimates. The use of the simplified solution to the elasticity problem does not make it possible to take into account this displacement.

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ERRATUM

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The paper contains a number of misprints:

1. Table 1 (p. 122) shows the incorrect value of the parameter ξ_π for KCl. The correct value is indicated in Table 1 of this paper and is used in the calculations.
2. On page 117, in the second paragraph of the left column, in a sentence beginning with the words ‘Soms and Tarasov also noted,’ instead of m3 crystals, we talk about m3m crystals.
3. On page 129 in the first paragraph of Section 5.3.1, instead of m3 crystals, we talk about m3m crystals.