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Method for calculating the optical axis position in laser cavities

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Abstract. **We present a method for calculating the optical axis contour in laser cavities using the geometric optics approximation. The deviation of the position of the optical axis in four-mirror nonplanar ring cavities is calculated for various cavity perturbations. Various types of laser cavities are considered and classified according to their symmetry. Deviations of the optical axis are determined in a nonplanar ring cavity that provides Kerr mode locking operation. A formula is given that relates the bend angle in four-mirror cavities to the angles of radiation incidence on the mirrors.**

Keywords: *optical axis, nonplanar cavity, ring cavity, laser gyro, Zeeman laser gyro, Kerr mode-locked laser.*

1. Introduction

The position of the optical axis in laser cavities determines both the direction of the output radiation and the stability of the power and transverse structure of the generated radiation, as well as a number of other important characteristics of the laser. Finding an optical axis contour, which in the general case can be a closed trajectory, is an urgent mathematical problem [1–3], but its solution is rather complicated in 3D space. For ring lasers used in gyros, knowledge of the optical axis displacement is of great importance for achieving high accuracy in measuring angular quantities.

Ring nonplanar cavities are used in laser gyros [4, 5] with a magnetooptical bias. To ensure high accuracy in a laser gyro, it is important to know not only the initial position of its optical axis contour, but also its displacement during operation. Consider symmetric four-mirror cavities. Let us introduce the xyz coordinate system so that the yz plane corresponds to the symmetry plane of the cavity, which includes its diagonal connecting opposite mirrors. The use of the Zeeman or Faraday effects to create a bias in such a gyro requires circular polarisation of radiation, which can be formed, e.g., by using a nonplanar cavity [6, 7]. To achieve high accuracy of a laser gyro, it is important to calculate the design of the cavity and estimate the position of its optical axis contour, including the case when one or more mirrors turn out to be not in the ideal (calculated) position, e.g., during the manufacture of the cavity or during the operation of the gyro. For stable operation of mode-locked lasers, it is especially important that the displacements of the position of the optical axis contour in the cavity, which arise when the position of its optical elements is changed, are small.

Determining the position of the optical axis contour in nonplanar cavities is a more complicated problem than in the case of planar ones, since the optical path is a 3D curve. Usually, ray matrices are used to calculate cavities [8–11]. For example, the calculation of planar nonlinear cavities was performed in [10], and the calculation of nonplanar cavities was performed in [12]. The authors of Ref. [12] used an individual coordinate system for each mirror and described the rotation of one coordinate system relative to another by a rotation matrix. In Ref. [10], planar cavities are considered in which the beam is not rotated. However, in a real case, the presence of the mirror tilts makes the majority of cavities nonplanar, and it is necessary to take into account the rotation of the planes of the beam incidence on the mirrors. For a nonplanar cavity, in which the beam is rotated upon reflection from each mirror, this situation can cause additional difficulties associated with allowance for the previously obtained deviations of the beam parameters from the calculated values both in the coordinate and in the direction due to the influence of other mirrors before the beam incidence. The deviations after reflection from one mirror must be recalculated taking into account the rotation of the axes for further calculation of the deviations after reflection from the next mirror. Considering deviations turns out to be difficult, especially if the mirror on which such radiation is incident is also tilted. When using ray matrices, these difficulties are not taken into account.

In the examples given in Refs [10, 12], the tilt of only one mirror is considered; therefore, the calculation of the beam deflections is simplified, and there are no cross terms in the mirror matrices responsible for the effect of the beam deflection along one axis on the deflection along the other, which is important for the cavity. These terms have the next order of smallness, and their influence can often be ignored. However, for cavities with spherical mirrors with small radii of curvature, as, e.g., in a laser with Kerr mode locking, the influence of the cross terms is significant. In this work, a unified coordinate system is introduced for the entire cavity. This allows solving problems related to the deflection of the mirrors and the beam along different axes; however, it is difficult to solve such a problem analytically and one has to use numerical methods for finding the contour, which requires a large amount of calculations and was earlier difficult.

2. Different types of four-mirror nonplanar cavities

Consider three types of cavities, each of them formed by four mirrors, separated by a free space. To calculate the cavity, it

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is necessary to specify the parameters that determine the position of the mirrors. The cavity is schematically shown in Fig. 1. The coordinate system used in the calculations is also shown there. Points O, P, R and S are the centres of the mirrors. Mirrors 1, 2 and 4 are flat and mirror 3 is spherical. Point O coincides with the origin of the coordinate frame. The vector defining the x axis comes out of the point O, which is a point of the mirror and in an ideal case belongs to the optical path. This vector lies in the plane of the mirror. The vector defining the y axis comes out of the same point and lies on the line OR, and so the angle between the vectors defining the x and y axes is 90°. The vector defining the z axis is directed upwards, and so the vectors defining the x, y, and z axes form the right-hand triple of vectors, with the angles between any two vectors being 90°. Since the mirror in a nonplanar cavity is placed at an angle, only the vector defining the x axis lies in the mirror plane. The centres of mirrors can be specified as points $X(x, y, z)$. The flat mirrors are described by the plane equation $A_i x + B_i y + C_i z + D_i = 0$, where $i = 1, 2, 4$. The spherical mirror is described by the sphere equation $(x - x_c)^2 + (y - y_c)^2 + (z - z_c)^2 = R^2$, where $X_c(x_c, y_c, z_c)$ is the centre of the sphere and R is its radius.

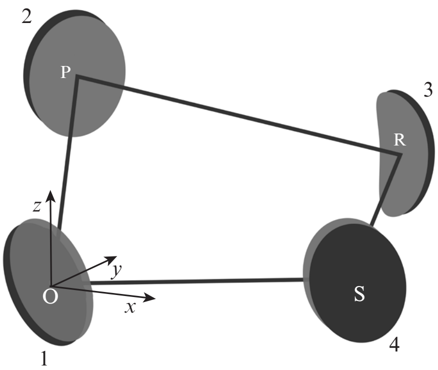

Figure 1. Schematic of a nonplanar four-mirror cavity.

Let us denote the half-diagonal lengths, i.e., the lengths of segments PS and OR, by W and L, and the distance between the diagonals by H. First, consider a cavity symmetric with respect to both the plane passing through the line OR orthogonally to PS and the plane passing through the line PS orthogonally to OR, where $W = L$. The centres of mirrors can be specified as O(0, 0, 0), P($-W, W, H$), R($0, 2W, 0$), and S(W, W, H). In the projection onto the xy plane (top view), the optical path looks like a square. We conventionally call such a configuration a cavity of the first type (Fig. 2a). For it, the coordinates of the sphere centre are

$$x_c = 0, \quad y_c = W t_s, \quad z_c = H t_s, \tag{1}$$

where $t_s = R/\sqrt{W^2 + H^2}$.

In the cavity of the next type, there is also symmetry with respect to both the plane passing through the line OR and the plane passing through the line PS. However, the dimensions of the cavity along different axes will be different and the coordinates of the points will be specified as O(0, 0, 0), P($-W, L, H$), R($0, 2L, 0$), and S(W, L, H), where $W \neq L$. The

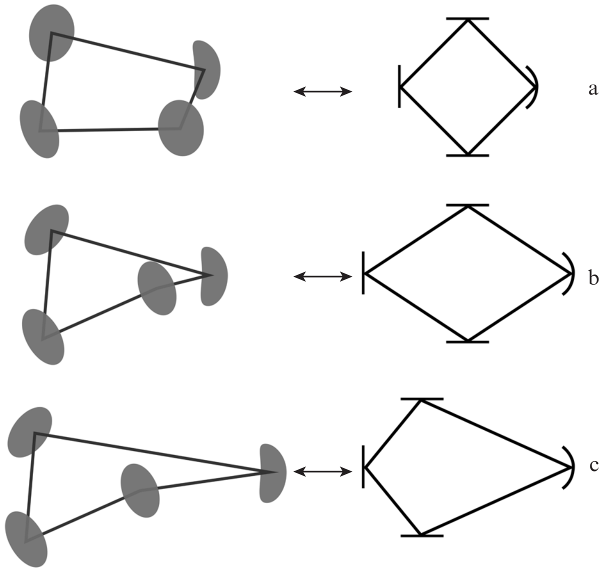

Figure 2. Optical schemes and top view of various types of cavities, shown in trimetric projection. The cavity schemes are based on numerical calculations.

top view of the optical path will be diamond-shaped. We will call such a cavity a cavity of the second type (Fig. 2b). The parameters of the mirrors are determined in the same way as for a cavity of the first type. As a result of the calculation, we obtain $A_1 = 0$, $B_1 = L$, $C_1 = H$, $B_2 = B_4 = 0$, $A_2 = W$, $A_4 = -W$, $C_2 = C_4 = -H$, and $D_1 = 0$, $D_2 = D_4 = W^2 + H^2$. For a spherical mirror

$$x_c = 0, \quad y_c = 2L - \sqrt{\frac{R^2}{1 + H^2/L^2}},$$
$$z_c = \sqrt{R^2 - (2L - y_c)^2}. \tag{2}$$

The cavity of the last of the considered types is symmetric only with respect to the plane passing through the straight line OR orthogonally to PS (Fig.2c). The points will be specified as O(0, 0, 0), P($-W, L_1, H$), R($0, L_2, 0$), and S(W, L_1, H), where L_2 is the cavity diagonal length, and coordinate L_1 defines the position of its second diagonal. However, in this case, mirrors 2 and 4 are not parallel to the plane passing through the OR axis, and so their parameters cannot be determined similarly to the previous cases. It is necessary to take into account the change in the beam direction when reflected from the mirror. For mirror 1, the calculation method is the same. As a result, we obtain $A_1 = 0$, $B_1 = L_1$, $C_1 = H$, and $D_1 = 0$.

For mirrors 2 and 4

$$A_2 = \frac{W}{2q_1} + \frac{W}{2q_2}, \quad B_2 = -\frac{L_1}{2q_1} + \frac{L_2 - L_1}{2q_2},$$

$$C_2 = -\frac{H}{2q_1} - \frac{H}{2q_2}, \quad D_2 = WA_2 - L_1 B_2 - HC_2,$$

$$A_4 = -A_2, \quad B_4 = B_2, \quad C_4 = C_2,$$

$$D_4 = -WA_4 - L_1B_4 - HC_4,$$

where

$$q_1 = \sqrt{W^2 + L_1^2 + H^2};$$
$$q_2 = \sqrt{W^2 + (L_2 - L_1)^2 + H^2}. \qquad (3)$$

For the spherical mirror, we obtain

$$x_c = 0,$$
$$y_c = L_2 - \frac{(L_2 - L_1)R}{\sqrt{(L_2 - L_1)^2 + H^2}}. \qquad (4)$$

The sensitive area, i.e., the area encompassed by the optical contour [13], for such cavities (top view) can be determined as $2W^2$, $2WL$, and WL_2 for the first, second and third type, respectively.

The mode separation for longitudinal modes can be calculated using the formula [5]

$$\nu_c^+ - \nu_c^- = \frac{c}{L_c}\frac{\rho_\Sigma}{\pi}, \qquad (5)$$

where $\nu_c^\pm$ are the cavity mode frequencies; L_c is the cavity perimeter; and ρ_Σ is the total angle of beam rotation per round trip, which for the cavities of the first, second and third type, respectively, is determined by the expressions

$$M(\mathbf{v},\theta) = \begin{vmatrix} \cos\theta + (1-\cos\theta)i^2 & (1-\cos\theta)ij - (\sin\theta)k & (1-\cos\theta)ik + (\sin\theta)j \\ (1-\cos\theta)ji + (\sin\theta)k & \cos\theta + (1-\cos\theta)j^2 & (1-\cos\theta)jk - (\sin\theta)i \\ (1-\cos\theta)ki + (\sin\theta)j & (1-\cos\theta)kj + (\sin\theta)i & \cos\theta + (1-\cos\theta)k^2 \end{vmatrix}. \qquad (11)$$

$$\rho_\Sigma = 4\arccos\left(\frac{W^2}{W^2 + H^2}\right), \qquad (6)$$

$$\rho_\Sigma = 4\arccos\left(\frac{WL}{\sqrt{W^2 + H^2}\sqrt{L^2 + H^2}}\right), \qquad (7)$$

$$\rho_\Sigma = 2\arccos\left(\frac{WL_1}{\sqrt{W^2 + H^2}\sqrt{L_1^2 + H^2}}\right) + 2\arccos\left(\frac{WL_2}{\sqrt{W^2 + H^2}\sqrt{L_2^2 + H^2}}\right). \qquad (8)$$

Let us present the formula for the dependence of the cavity bend angle on the angles of radiation incidence on the mirrors. It is valid for cavities of the first and second types. The bend angle is the angle between opposite planes, and so there are two of them:

$$\cos\alpha = \frac{-\cos^2\gamma_2 + \sqrt{(\cos^2\gamma_1 - \cos^2\gamma_2)^2 + 4\cos^2\gamma_1\cos^2\gamma_2\cos^2\gamma}}{\cos^2\gamma_1}, \qquad (9)$$

where α is the bend angle, γ is the angle between the planes of the beam incidence on adjacent mirrors, and γ_i is the angle of beam incidence for each pair of opposite mirrors ($i = 1, 2$); for both mirrors it will be the same. In order to get another bend angle, it is necessary to interchange indices 2 and 1 everywhere.

For example, in a cavity of the first type with a perimeter of 198 mm, where the rotation of the radiation beam per round trip is 90° (in this case, the spectrum is equidistant), the side length is determined by the expression

$$l = \sqrt{2W^2 + H^2}. \qquad (10)$$

Using Eqns (6) and (10), we obtain $W = 34.3$ mm and $H = 9.85$ mm.

3. Calculation of the parameters for mirrors in an imperfect position

Next, we calculate a cavity in which the mirrors are not in an ideal (calculated) position. Let us show how the tilt and displacement of the mirrors from their calculated positions can be taken into account.

Flat mirrors are described using the plane equation, and so one can describe their displacements by changing the coefficients in the equation. The normal vector determines the position of the mirror; therefore, to take into account the rotation of the mirror, it is necessary to multiply the matrix of rotation about the axis by the normal vector.

Let the axis of rotation be specified by the unit vector $\mathbf{v}(i, j, k)$, and the angle of rotation be θ. Then the rotation matrix in the Cartesian coordinates has the form

Consider, e.g., the rotation of mirror 1 in a resonator of the first type around the x axis by angle θ. The coordinates of the normal vector of this mirror can be described as

$$\begin{vmatrix} A \\ B \\ C \end{vmatrix} = \begin{vmatrix} 1 & 0 & 0 \\ 0 & \cos\theta & -\sin\theta \\ 0 & \sin\theta & \cos\theta \end{vmatrix} \begin{vmatrix} 0 \\ W \\ H \end{vmatrix} = \begin{vmatrix} 0 \\ W\cos\theta + H\sin\theta \\ -W\sin\theta + H\cos\theta \end{vmatrix}. \qquad (12)$$

To allow for the displacement of the mirror perpendicular to its plane, it is necessary to change the parameter D.

To allow for the displacement of a spherical mirror, it is necessary to shift the centre of the sphere. To describe the displacement of the spherical mirror in the mirror plane, it is necessary to change the coordinates x_c and z_c, while the coordinate y_c remains unchanged.

To describe the displacement of a spherical mirror parallel to itself, it is necessary to use the formulae

$$x_c' = x_c,\ z_c' = z_c \pm \frac{d|z_c|}{R},\ y_c' = y_c \pm \frac{d(p - y_c)}{R}, \qquad (13)$$

where d is the distance over which the displacement occurs; and $p = 2W, 2L, L_2$ for resonators of the first, second and third type, respectively. The choice of the sign depends on the displacement direction.

To describe the rotation of a mirror in different planes, it is necessary to use formulae that take into account the rotation of a spherical mirror at an angle θ in the plane of incidence:

$$z_c' = z_c,\ x_c' = x_c \pm R(1-\cos\theta)\frac{p-y_c}{R},$$
$$y_c' = y_c \pm R\sin\theta\frac{p-y_c}{R}. \tag{14}$$

In the sagittal plane, these formulae have the form

$$x_c' = x_c,\ y_c' = y_c + g\sin(\theta/2),$$
$$z_c' = z_c \pm g\cos(\theta/2), \tag{15}$$

where

$$g = \sqrt{2R^2 - 2R^2\cos\theta}. \tag{16}$$

This describes the displacement and deviation of any mirrors from their ideal position.

4. Finding the optical path contour

Now that the model of the cavity has been developed, we will find the optical path contour in it. First, it is necessary to calculate the passage of the beam in the cavity based on the laws of geometric optics [14]; in the future, such a calculation will be used to determine the contour. The cavity consists of four reflective elements with a free space between them. Then the ray between the mirrors can be described using a straight line to which the points of the mirror $X_i(x_i, y_i, z_i)$ belong, where i is the number of the mirror, and the direction of the ray propagation is given by the vector $\boldsymbol{a}(a_x, a_y, a_z)$. Therefore, knowing the point on the mirror and the direction of the beam propagation, one can find the point on the next mirror. For this purpose, it is necessary to determine the point of intersection of a straight line and a plane (for a flat mirror) or a sphere. The positions of the flat mirrors are specified by the planes described in 3D space by the equations $A_ix + B_iy + C_iz + D_i = 0$ with the normal vector $\boldsymbol{n}_i(A_i, B_i, C_i)$, where $i = 1, 2, 4$. The position of the spherical mirror ($i = 3$) is determined by the sphere equation $(x - x_c)^2 + (y - y_c)^2 + (z - z_c)^2 = R^2$. The coordinates of the point on the mirror number $i + 1$, which the ray hits, are calculated using the formulae (for convenience, we consider the points X_i and X to be vectors with three components)

$$\boldsymbol{X}_{i+1} = \boldsymbol{X}_i + s\boldsymbol{a},$$
$$s = \frac{A_{i+1}x_i + B_{i+1}y_i + C_{i+1}z_i + D_{i+1}}{A_{i+1}a_x + B_{i+1}a_y + C_{i+1}a_z} \tag{17}$$

for a flat mirror and

$$\boldsymbol{X}_{i+1} = \boldsymbol{X}_i + s\boldsymbol{a},\ s = \frac{-b + \sqrt{b^2 - 4ah}}{2a},$$
$$a = |\boldsymbol{a}|^2,\ b = 2\boldsymbol{X}_i\boldsymbol{a} - 2\boldsymbol{X}_c\boldsymbol{a},$$
$$h = |\boldsymbol{X}_i|^2 - 2\boldsymbol{X}_i\boldsymbol{X}_c + |\boldsymbol{X}_c|^2 - R^2 \tag{18}$$

for a spherical mirror.

The ray vector after reflection can be found using the formula

$$\boldsymbol{a}_{i+1} = \boldsymbol{a}_i - 2\frac{(\boldsymbol{a}_i\boldsymbol{n}_{i+1})\boldsymbol{n}_{i+1}}{|\boldsymbol{n}_{i+1}|^2}, \tag{19}$$

where for a spherical mirror

$$\boldsymbol{n}_{i+1} = \boldsymbol{X}_{i+1} - \boldsymbol{X}_c.$$

Similar formulae are given in Ref. [15]. Next, we proceed to determining the position of the optical contour. On mirror 1, the beam will have parameters $\boldsymbol{a}_1$ and $\boldsymbol{X}_1$. Let us calculate its passage through the cavity using Eqns (17)–(19). As a result, after reflection from mirror 1, we have a ray with parameters $\boldsymbol{a}_5$ and $\boldsymbol{X}_5$. According to the optical contour definition [5], the calculated path will be an optical contour if $\boldsymbol{a}_1 = \boldsymbol{a}_5$ and $\boldsymbol{X}_1 = \boldsymbol{X}_5$. The analytical solution of this system consists of a large number of intermediate expressions, and they are composed iteratively, which makes the solution of the equations rather complicated. The analytical solution is seen to be a difficult task, and so numerical methods can be used. We set the parameters of the initial beam and find the parameters after each round trip of the cavity. Changing the parameters (coordinates and direction), we search for such a ray, the parameters of which, after the round trip, would differ as little as possible from the initial ones on the same mirror. Since the deflection is calculated on the mirror, if the equation of the mirror plane and two coordinates are known, one can always find the third coordinate; similarly, by normalising the ray vector, one can determine its third component. Therefore, it is sufficient to change four rather than six quantities. Thus, we make a round trip, fix the beam parameters, then change the parameters of the initial beam, find the differences $\boldsymbol{a}_1 - \boldsymbol{a}_5$ and $\boldsymbol{X}_1 - \boldsymbol{X}_5$, compare them with the previous ones and obtain such an initial beam for which these differences would be as small as possible. This is how we find the desired ray.

5. Calculations and measurements

To check the efficiency of the method, the calculations of the optical contour were carried out and their results were compared with the experiment. Consider cavities of the first type described above. For such cavities, the deviations of the positions of the mirrors from their calculated values and the distance between the mirrors are known. Considering these data, the optical path contour in the cavity was calculated, after which the resulting contour was compared with that for the cavity with the mirrors in the ideal position.

The experimental measurement of the optical path position was carried using a setup (Fig. 3) including a master ring He–Ne laser (with a wavelength of 632.8 nm) with a nonplanar cavity frequency-tunable by means of piezo motors, the studied nonplanar cavity of a laser gyro, as well as a laser control unit and measuring instruments with a computer. To obtain the transverse structure of the beam in the region of the aperture, a CCD camera with a special objective was used. When measuring the mode formed in the cavity, it was frequency tuned to resonance with a particular mode of the master laser and appropriately oriented relative to the incident radiation beam.

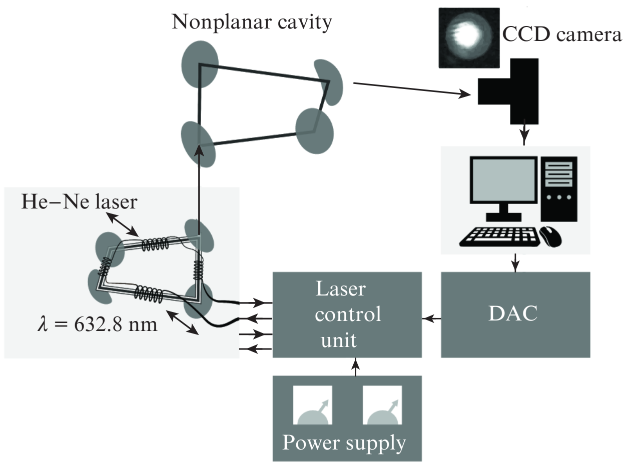

Figure 3. Experimental setup.

Finally, the mode formed in the cavity was recorded by a CCD camera (Fig. 3). The linear dimensions were determined by comparison with the diameter of the channels that form the cavity of the laser gyro, obtained earlier from other measurements.

The results of calculations of the optical contour displacement on the mirror using numerical simulation, as well as the experimentally measured displacements, are given in Table 1. The first two columns show the deviations of the mirror orientations and the lengths of the channels forming the cavity, and the last one shows the calculated and experimental displacements of the optical contour (absolute values).

The results of the experimental measurements and calculations show (Table 1) that the method presented in this work allows determining the position of the optical contour in the cavity with good accuracy. It can be used both in the manufacture of laser gyros and for evaluating the laser gyro errors and changes in its operation, associated with the contour displacement.

Table 1.

Cavity number	Angular deflections of mirrors $\delta\varphi_1, \delta\varphi_2, \delta\varphi_3, \delta\varphi_4$	Linear deviations of channel lengths $\Delta l_1, \Delta l_2, \Delta l_3, \Delta l_4$ /μm	Calculated (measured) displacement of the optical contour/μm
1	−1′58″, 1″, −2′, 44″	1.5, 2.3, 17.1, 38.5	83 (102)
2	65″, −1′15″, 34″, −56″	18, 5.4, −14, −28	96 (93)
3	−18″, 42″, −29″, 50″	25, 3, 44.1, 12.5	80 (78)
4	1′18″, 22″, 57″, 14″	7.5, −5.3, 22.3, 8.5	110 (115)

6. Examples of using the method

The method was used to estimate the position of the optical contour under in-phase (in one direction) displacement of mirrors 2 and 4 (Fig. 4). Calculations have shown that the optical path contour is also shifted in parallel without other distortions.

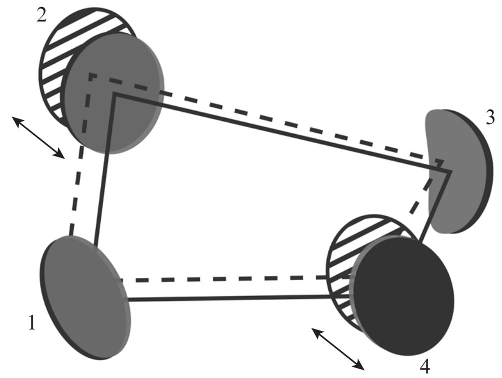

Figure 4. Displacement of the optical contour upon in-phase movement of the mirrors.

In a cavity of the first type, the in-phase displacement of the mirrors located diagonally (2 and 4) leads to a parallel translation of the entire contour without deforming it and changing its dimensions if all the mirrors are flat. If the cavity contains a spherical mirror in the orthogonal diagonal, then the changes in the arm lengths are very small. The antiphase displacement of these mirrors will cause a shift of the contour up or down and, therefore, increase or decrease its size.

The proposed method was used to calculate the optical contour and estimate its displacement caused by deflections of the mirrors in cavities of the first (Fig. 2a) and third (Fig. 2c) types. The results of the cavity calculations are shown in Fig. 5, which presents the dependences of the coefficient $K = \Delta L/\beta$, which characterises the displacement of the optical contour caused by deflections of the mirrors, on the angle of rotation of the beam cross section per round trip of the cavity for cavities of the first and third type. All calculations were performed for cavities in which the radius of the spherical mirror curvature was 3 m. ΔL is the maximum displacement of the optical contour in the cavity. The choice was made from four displacements on the mirrors, but since the beam propagates along a straight line, the maximum displacement will be on the mirrors rather than between them. Thus, we estimate the maximum displacement in the entire cavity. The angle β is the deflection angle of the flat mirror to which the piezo motor is attached. In the calculations, the sensitive area and scale factor were the same for all cavities, which is important when comparing laser gyros. The scale factor was 3 deg h^{-1} Hz^{-1}.

It can be seen from Fig. 5 that in cavities of the first type [curve (*1*)], the same deflections of the mirrors lead to a smaller displacement of the optical contour than in cavities of the third type [curve (*2*)]. Note that the general form of the dependences agrees with the results of Ref. [12].

Kerr mode locking in femtosecond lasers based on broadband active media $YAG:Cr^{4+}$ and $Al_2O_3:Ti^{3+}$ requires strong

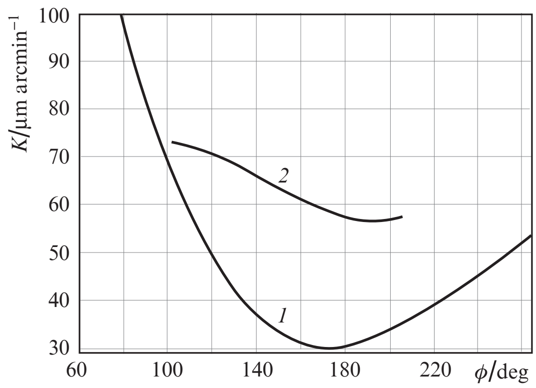

Figure 5. Dependences of the coefficient K, characterising the displacement of the optical contour caused by deflections of the mirrors, on the angle of rotation ϕ of the beam cross section per cavity round trip for cavities of the first (*1*) and third (*2*) types.

focusing of radiation and a very stable position of the beam in the aperture region with respect to external perturbations. Therefore, here we performed calculations of the beam displacement for various deflections of the cavity elements.

We considered a virtually planar ring cavity with small deviations from planarity, consisting of two flat mirrors (1 and 5), two spherical mirrors (2 and 3), and one prism (4) (Fig. 6). The distance is 420 mm between mirrors 1 and 2 and 96 mm between mirrors 2 and 3. The angles of incidence are 22.5° on mirrors 2 and 3, and 15° on mirrors 1 and 5.

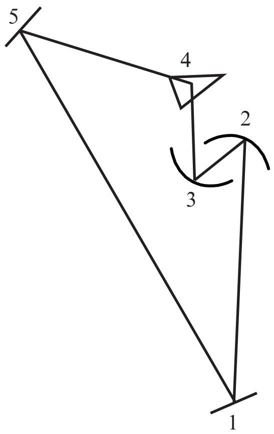

Figure 6. Schematic of a ring cavity providing tight focusing of radiation in an active medium to obtain Kerr mode locking.

In the cavity, the flat (1) and spherical (2) mirrors were simultaneously tilted. As a result, the optical contour was found and its position on the flat mirror 5 was determined. The flat mirror was tilted all the time by a fixed angle, which did not change. The calculation results are shown in Fig. 7, where each point corresponds to the position of the optical contour on the plane mirror 5 with the simultaneous tilting of the plane mirror in the plane of the cavity and the spherical mirror perpendicular to the plane of the cavity.

It can be seen from Fig. 7 that the displacements of the optical contour have a complex nonlinear character, which is difficult to reflect in two-dimensional space. In the course of calculations, it was found that the tilt of the flat mirror of the

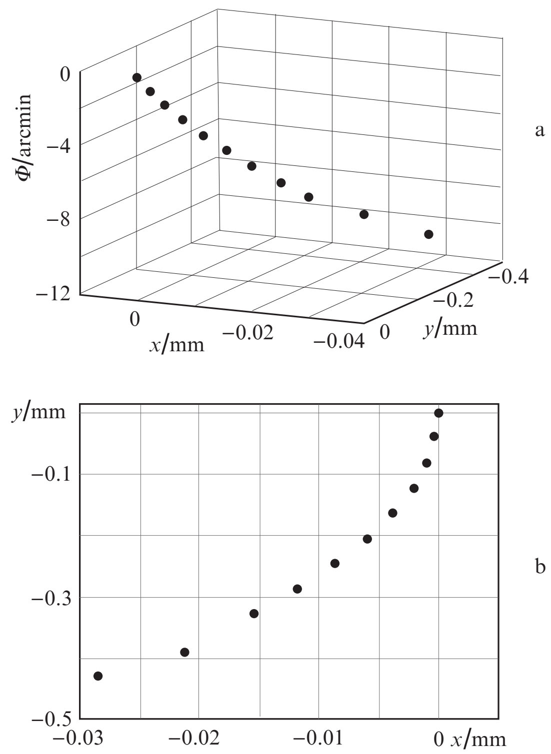

Figure 7. Displacement of the optical contour position on a flat mirror when two mirrors are tilted, calculated for a cavity providing strong focusing of radiation in an active medium with Kerr mode locking, versus the tilt angle Φ of a spherical mirror in space (a) and in the mirror plane (b).

opposite arm affects the displacement of the optical contour much weaker than the tilt of the spherical one. Figure 7 shows the results of calculations of the optical contour in the case when the spherical mirror is tilted and the flat one is not. The results of experimental studies of the optical contour displacement in a YAG:Cr^{4+} ring laser with Kerr mode locking showed good agreement with the results of calculations. Thus, to obtain stable Kerr mode locking in a laser with such a cavity, high stability and a more accurate initial positioning of the spherical mirrors are required.

7. Conclusions

We have described a method for calculating the optical path contour, which makes it possible to determine its position with high accuracy when several mirrors are tilted simultaneously. This is important, for example, for calculating the optical contour in cavities with strong radiation focusing, including those in lasers with Kerr mode locking, where spherical mirrors have small radii of curvature. In methods based on ray matrices, this would require the inclusion of cross terms, which complicates the calculations. Overall, the use of geometric optics in our proposed approximation method makes the entire calculation process clear and understandable. Symmetric four-mirror nonplanar ring cavities were considered and their classification was carried out. Expressions for calculating the position of mirrors in such cavities are presented. It was found that in cavities of the first type, the displacement of the optical path contour is less than in cavities of the third type, at the same tilts of the mirrors, i.e., they are more resistant to external perturbations if we consider cavities in which the magnitude of the scale factor is preserved. In fact, the method for calculating the position of the optical path contour can be used to calculate cavities with different numbers of mirrors. Moreover, we obtained an expression that relates the bend angle for a four-mirror nonplane cavity and the angles of incidence of radiation on the mirrors.

The major advantages of the considered method for determining the optical contour will appear when calculating nonplanar cavities. It is shown that in a symmetric nonplanar cavity the in-phase displacement of the mirrors located diagonally leads to a parallel translation of the entire contour without deforming and changing its dimensions, if all mirrors are flat, and with very small changes in the arm lengths in the presence of a spherical mirror in the orthogonal diagonal. The antiphase displacement of these mirrors leads to an upward or downward displacement of the contour and, consequently, to an increase or decrease in its dimensions.

Calculations of the cavity for a laser with Kerr mode locking are performed. The configuration is determined that has the best stability of the optical axis position in the region of the aperture, which provides modulation of losses during Kerr mode locking. It is shown that the displacement of the contour on the output mirror in such a cavity when the spherical mirror is deflected is much larger than when the plane mirror is deflected in the opposite arm.

Designing a laser gyro requires high-precision cavity manufacturing, and the method developed by us that allows calculating the displacement of the cavity optical path with the manufacturing imperfections taken into account, is undoubtedly important for reducing errors in the operation of laser gyros.

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