

https://doi.org/10.1070/QEL17617

Breaking of a nonlinear wake wave excited by a laser pulse during its interaction with a semi-infinite plasma

S.V. Kuznetsov

Abstract. **Analytical methods are used to study the properties of a wake wave excited by a relativistic laser pulse passing through a blurred boundary of a homogeneous plasma. It is found that on the plasma density plateau near its boundary, the wake wave is not regular and its phase velocity depends on the spatial coordinate. It is shown that the breaking of a wake wave is a threshold process in terms of the electron oscillation energy; the influence of the transition layer parameters on the process of wake wave breaking is determined.**

Keywords: *laser pulse, wake wave, phase velocity, electron oscillations, inhomogeneous plasma.*

1. Introduction

In recent decades, a significant progress has been observed in the development of the technology of laser-plasma acceleration of electrons, based on which electron bunches with energies of several GeV have already been obtained [1]. This technology is promising for developing compact facilities capable of producing high-energy electron bunches required for many applications. However, the development of laser-plasma accelerators suitable for practical use still faces a number of problems associated with the lack of stability during the acceleration of electron bunches and poor control over their characteristics. These problems largely come to the fore even at the stage of injection of electrons into the accelerating plasma wave produced by a laser pulse. This is because in a laser-plasma accelerator, a bunch of accelerated electrons is usually formed from background plasma electrons upon the breaking of the wake wave; the process by its nature often depends on many random factors and is therefore difficult to control.

There are methods for controlled injection of electrons, for example, injection of electrons into the wakefield due to the collision of two laser pulses [2] or injection of electrons due to ionisation [3], but they complicate the design of the accelerator or cause a number of other problems. For this reason, the injection of electrons during the breaking of the laser pulse wake wave remains the most frequently used method of injecting electrons in a laser-plasma accelerator. This method requires a deeper study, both theoretical for better understanding the physical mechanism under various conditions and practical in order to develop injection control methods.

S.V. Kuznetsov Joint Institute for High Temperatures, Russian Academy of Sciences, ul. Izhorskaya 13, stroenie 2, 125412 Moscow, Russia; e-mail: svk-IVTAN@yandex.ru

Received 7 July 2021
Kvantovaya Elektronika **51** (9) 819–825 (2021)
Translated by V.L. Derbov

Among the studies in this direction carried out in Russia, one should distinguish Refs [4, 5], which laid the foundation for the study of the process of self-injection of electrons into the wake wave of a laser pulse during its propagation in an inhomogeneous plasma. In these works, Bulanov et al. showed that on a decreasing plasma density gradient, the phase velocity of the wake wave decreases, and it becomes possible to trap electrons in the accelerating field of the wake wave. It was also shown that the characteristics of the bunch could be controlled by choosing the plasma density gradient and laser pulse parameters.

Among the works of foreign researchers, special attention should be paid to paper [6], which reports the results of studying the process of self-injection of electrons into the wakefield of a laser pulse propagating in the plasma bubble regime in a gas cell with uniform electron density. The authors presented the results of experiments and simulations, from which it follows that the trapping of electrons into the accelerating field of the cavity actually occurs in two stages: at the beginning, when the self-focusing of the laser pulse breaks the wake wave, electrons are captured into the bunch formed for acceleration by the longitudinal self-injection mechanism, and then due to the transverse one. It was found that longitudinal injection, which is always observed at the beginning of the bunch formation process, leads to a much more stable acceleration and the formation of better-quality electron beams.

This conclusion is also confirmed by the results of Ref. [7], in which, during numerical simulations, a controlled injection of electrons into the wake field produced by a laser pulse was observed in a distinctly defined place in space at a definite time instant. This study examined the passage of a laser pulse through a rarefied plasma target with an up-ramp density profile followed by a plateau. It was shown that the large diameter of the laser focal spot leads to a substantially one-dimensional wakefield formation regime, which differs from the bubble regime that occurs for tightly focused beams of the laser driver. Thus, conditions were provided for the longitudinal mechanism of self-injection of electrons, and the ascending profile led to a sharp breakdown of the one-dimensional wave at the rise-plateau transition. The results of the study showed that under these conditions an ultra-thin (several nanometres, which corresponds to an attosecond duration) ultradense relativistic electron layer is generated, which is injected and accelerated in the wake field.

Works [8–12] is devoted to the theoretical analysis of the process of longitudinal self-injection of electrons into a wake wave generated by a laser pulse. They show that this phenomenon is based on the process of crossing the trajectories of electrons performing longitudinal oscillations under the action of a laser pulse. The intersection of electron trajectories leads to the breaking of the wake wave, mixing of electrons and their trapping in the accelerating field of the wake wave. Thus, the breaking of the wake wave and the generation of electron bunches by a laser pulse are interrelated processes. A theoretical study of the conditions under which the breaking of a wake wave is possible in the case of a laser pulse propagating through an inhomogeneous plasma provides a key to understanding the mechanism of electron bunch generation under various conditions and to optimising this process.

In the general case, the phenomenon of the generating electron bunches by a laser pulse is a very complex process that strongly depends on the plasma profile, its parameters, and the characteristics of the laser pulse. In this paper, we investigate a rather limited problem in which a laser pulse propagates along an ascending plasma profile, and we study its solutions allowing the generation of electron bunches and passing in the limiting cases to the solutions obtained earlier in [8–12] for a plasma with a sharp boundary. In practice, this means that in our study, a transition layer of greater or lesser extent replaces the sharp plasma boundary and the behavior of the wake wave for a laser pulse of relativistic intensity interacting with such a semi-infinite plasma is investigated. The study aim is to find out the properties of the wake wave in such conditions, which in this case turns out to be significantly irregular and unsteady, and to determine the conditions for its breaking.

2. Statement of the problem

Let us consider in one-dimensional geometry the process of penetration of a circularly polarised laser pulse of relativistic intensity into a semi-infinite homogeneous rarefied plasma with a transition layer at the boundary separating it from vacuum. Let the laser pulse propagate along the normal to the plasma boundary and assume that the ionic component of the plasma is stationary. Plasma electrons interacting with a laser pulse are initially displaced in the direction of its propagation, and then, returning back to their equilibrium point, begin to perform longitudinal oscillations around it, thereby forming a wake wave.

Let us assume that the laser pulse is short enough so that the near-boundary electrons, which were initially at the plasma density plateau, leave the laser pulse before they can pass into the transition layer during their motion. This assumption is fulfilled if the length of the laser pulse does not exceed the amplitude of the electron oscillations. We will also assume that when a laser pulse propagates along the plasma density plateau, its group velocity and shape do not change. Under these conditions, all the electrons that were initially on the plasma density plateau, immediately after the end of the action of the laser pulse on them, will move along trajectories that are similar, regardless of the initial location of the electrons on the plateau. The difference between the trajectories is expressed only in a certain phase shift associated with the fact that the electrons located farther from the plasma boundary will later interact with the laser pulse.

The initial similarity of the trajectories of electrons opens up the possibility of studying the remote consequences of their motion without a complete knowledge of all the characteristics of the laser pulse that excited this motion. Thus, the motion of an oscillating electron near its centre of oscillation, which is its initial position on the plateau, is completely determined by the value of the total energy of such a plasma oscillator, obtained by it from the laser pulse, and the initial phase of this motion, which is set by the group velocity of the laser pulse $V_{\rm gr}$ on the plateau.

3. Trajectories of plasma electrons after interaction with a laser pulse

Let us choose the z axis in the direction along which the laser pulse propagates, and let the origin on this axis be the point coinciding with the beginning of the plasma density plateau, that is, the point $z = 0$ is on the edge of the transition layer bounding a homogeneous plasma. The plasma density profile prior to exposure to a laser pulse (and, accordingly, the profile of the stationary ionic background) is given by the dependence $n(z)$, which on the plateau at $z \geqslant 0$ has a constant value $n(z) = n_0$. The displacement of the electron from the point of its initial location z_0 to the point z gives rise to a charge separation field E_z, which returns the electron back to its centre of oscillation:

$$E_z = 4\pi |e| \int_{z_0}^{z} n(z')\,\mathrm{d}z', \qquad (1)$$

where e is the electron charge. Note that an electron from the plasma density plateau can exit into the transition layer at the plasma boundary; therefore, Eqn (1) uses a general representation for the background profile of ion density.

The motion of the plasma oscillator obeys the energy conservation law:

$$\sqrt{m^2c^4 + p^2c^2} + 4\pi e^2 \int_{z_0}^{z} \mathrm{d}z' \int_{z_0}^{z'} n(z'')\,\mathrm{d}z'' = W_{\rm os}, \qquad (2)$$

where m is the electron mass; c is the speed of light; $p = mu \times (\sqrt{1 - u^2/c^2})^{-1}$ is the relativistic momentum of the electron; and $u = \mathrm{d}z/\mathrm{d}t$ is its velocity.

Relation (2) allows the trajectory of any electron from the plasma density plateau to be expressed in integral form. Let us assume that at the beginning of the motion after interaction with the laser pulse, the trajectories of all electrons from the plateau are similar. Then, taking into account the phase shift $\Delta z_0/V_{\rm gr}$ between the trajectories of electrons, whose centres of oscillations are at a distance of Δz_0 from each other, we can generally express the set of these trajectories depending on time t as

$$ct = \frac{cz_0}{V_{\rm gr}} + I(z, z_0, z_0) + KcT_{\rm lft}(z_0) + (K-1)cT_{\rm rgt}, \qquad (3)$$

where

$$I(z_1, z_2, z_0) = \int_{z_2}^{z_1} \mathrm{d}z' / \sqrt{1 - 1/F^2(z', z_0, W_{\rm os})}; \qquad (4)$$

$$F(z', z_0, W_{\rm os}) = \left(W_{\rm os} - 4\pi e^2 \int_{z_0}^{z'} \mathrm{d}z'' \int_{z_0}^{z''} n(z''')\,\mathrm{d}z'''\right) / (mc^2); \qquad (5)$$

$$T_{\rm rgt} = T_{\rm h} = 2c^{-1} \times \int_0^{A_{\rm m}} \mathrm{d}z' / \sqrt{1 - m^2c^4/[W_{\rm os} - 2\pi e^2 n_0 z'^2]^2}$$

is the time interval required for the electron to move from its oscillation centre z_0 to its extreme rightmost position $z_0 + A_m$ and return back; and $T_{lft}(z_0) = c^{-1}2I(z_0, z_{bn}(z_0), z_0)$ is the time interval required for the electron to move from the centre of oscillation z_0 to its extreme leftmost position $z_{bn}(z_0)$ and return back. The coordinate $z_{bn}(z_0)$ is the turning point on the electron trajectory, and its value must be determined from Eqn (2), since it is its root.

The difference in the expressions for electron oscillations to the left and to the right from its oscillation centre is due to the fact that electrons from the plasma density plateau when moving to the left can enter the transition layer, whereas when moving to the right from the oscillation centre, they have the same deviation amplitude $A_m = \sqrt{(W_{os} - mc^2)/(2\pi e^2 n_0)}$. When integrating Eqn (2), the integration constant is determined from the condition that the electron, initially located at the edge of the plasma density $z_0 = 0$ at the time $t = 0$, moves to the left with a velocity $u = c\sqrt{1 - m^2c^4/W_{os}^2}$. An integer $K = 1, 2, ...$ means the number of oscillations made by an electron with the oscillation centre at the point z_0, while Eqn (3) describes those parts of the trajectories of electrons in which they move from the leftmost point of their trajectory to the right. It is important to note that Eqn (3) is exact as long as the order of the electrons remains unchanged in the process of their oscillations.

The collective motion of plasma electrons after their interaction with the laser pulse forms a wake wave. When the amplitude of the laser pulse is not too high and the electrons under consideration do not leave the transition layer, a regular nonlinear wake wave is formed behind the laser pulse, which has the properties of an ordinary one-dimensional relativistic plasma wave [13]. This wave motion of the plasma is stationary in the coordinate system moving with the wave propagation velocity V_{ph}, which coincides with the propagation velocity of the laser pulse in a homogeneous plasma, $V_{ph} = V_{gr}$.

It is known that with an increase in amplitude, even in a homogeneous plasma, the plasma wave is broken [5, 13]. The breaking condition is the coincidence of the maximum electron oscillation velocity with the phase velocity of the wake wave, $u_{max} = V_{ph}$. Physically, the breaking of a plasma (wake) wave is expressed in the intersection of electron trajectories and the development of the process of generation of electron bunches [8–11].

The presence of a transition layer near the plasma boundary complicates the nature of the wake wave, because in the region of the plasma boundary, the wave becomes irregular. This is because when an electron passes through the transition layer, the similarity of the electron trajectories is violated, and, therefore, the phase difference between the electron oscillations changes. Moreover, it is known that it is precisely near the plasma boundary that electron bunches are generated, which are subsequently trapped and accelerated by the wake wave.

4. Threshold character of the process of wake wave breaking

In order to clarify the conditions for breaking the wake wave, leading to the generation of electron bunches, it is necessary to determine the conditions under which the trajectories of two neighbouring plasma electrons intersect. In this case, it is of particular importance to determine the minimum energy of longitudinal oscillations of electrons, at which the processes under consideration become possible in principle.

The condition for the intersection of the trajectory of an electron with the centre of oscillation at the point z_0 with the trajectory of an electron adjacent to it is $dZ/dz_0 = 0$, where $Z = Z(z_0, W_{os}, t)$ is its trajectory. Differentiating expression (3), we arrive at an equation that determines the coordinate z_{cr}, in which, for a given value of the total energy of longitudinal oscillations of electrons W_{os}, the trajectories will intersect:

$$\frac{c}{V_{gr}} - \frac{1}{\sqrt{1 - 1/F^2(z_{cr}, z_0, W_{os})}} - k_p^2 \int_{z_0}^{z_{cr}} \frac{\int_{z_0}^{z'} [(n(z_0) - n(z''))/n_0] dz''}{[F^2(z', z_0, W_{os}) - 1]^{3/2}} dz' + cK \frac{\partial T_{lft}(z_0, W_{os})}{\partial z_0} = 0, \quad (6)$$

where $k_p = \omega_p/c$, and $\omega_p = \sqrt{4\pi e^2 n_0/m}$ is the plasma frequency.

Of interest is the minimum oscillation energy of electrons, at which the intersection of their trajectories is possible. For an electron with an oscillation centre at the point z_0, the corresponding value of W_{os} is found from Eqn (6) according to the condition $dW_{os}/dz_{cr} = 0$. As a result, we arrive at the relation $z_{cr} = z_0$, that is, the intersection of the electron trajectory with the 'neighbouring' trajectory at the minimum possible value of its oscillation energy occurs at the time when the electron passes through its oscillation centre. Then, from relation (6), the value of the minimum energy for an electron with an oscillation centre at the point z_0 is determined:

$$W_{os} = mc^2 \Big/ \sqrt{1 - \frac{\beta^2}{\left[1 + \beta cK \dfrac{\partial T_{lft}(z_0, W_{os})}{\partial z_0}\right]^2}}, \quad (7)$$

where $\beta = V_{gr}/c$.

Analysis of Eqn (7) allows us to draw a number of important conclusions. Since for an electron entering the transition layer, the half-period of oscillations $T_{lft}(z_0, W_{os})$ at the leftmost point of the trajectory at a fixed value of W_{os} is the larger, the closer its oscillation centre is to the edge of the transition layer, for such electrons the derivative is $\partial T_{lft}(z_0, W_{os})/\partial z_0 < 0$. This means that the breaking in the first period of oscillations will occur at a lower energy than in subsequent periods and, therefore, in what follows, we take $K = 1$.

It also follows from Eqn (7) that the process of wake wave breaking at its first period in the interaction of a laser pulse with a semi-infinite plasma having a blurred boundary has a threshold character. This means that the intersection of the trajectory of an electron with a neighbouring one is possible only if its total energy of longitudinal oscillations exceeds a certain value determined in this case by Eqn (7). However, the value of W_{os} found in this way depends on the choice of an electron with the corresponding oscillation centre z_0. It is necessary to find the lowest energy value at which the wake wave can be broken somewhere in space, and to determine the position of the electron from which this process will begin.

To this end, consider the dependence of the time interval $T_{lft}(z_0, W_{os})$ on the coordinate z_0. The study shows that in the range $0 \leqslant z_0 \leqslant A_m$ the function $\partial T_{lft}(z_0, W_{os})/\partial z_0$ monotonically increases to a value equal to zero at the point $z_0 = A_m$, and then $\partial T_{lft}(z_0, W_{os})/\partial z_0 = 0$ for $z_0 \geqslant A_m$. According to

Eqn (7), this means that the threshold energy value for the process of breaking a wake wave in a plasma with a diffuse boundary is

$$W_{\rm os\,th} = mc^2/\sqrt{1-\beta^2} = mc^2\gamma_{\rm ph}. \qquad (8)$$

Note that the value of the threshold energy does not depend on the shape of the transition layer and, accordingly, coincides with the value obtained in [8] for the case of a sharp plasma boundary. The position of the electron, from which the process of breaking the wake wave begins, $z_0 = A_{\rm m}$, also coincides with the result of [8], since this electron is ahead of the oscillation phase of other electrons with oscillation centres $z_0 > A_{\rm m}$, for which the threshold energy is the same.

To illustrate these conclusions, we calculated the penetration of a circularly polarised laser pulse of relativistic intensity into the plasma. At the moment t_0 the pulse passing the beginning of the plasma plateau ($z = 0$) has an envelope of the form $a = a_0\cos^2[(t-t_0)/\tau_{\rm las}]\theta(\pi\tau_{\rm las}/2 - |t-t_0|)$. Here $a_0 = |e|A_0/mc^2 = 5.652$ is the dimensionless amplitude of the vector potential; $\tau_{\rm las}$ is the laser pulse duration corresponding to its FWHM duration $\tau_{\rm FWHM} = 1.143\tau_{\rm las} = 12$ fs; and θ is the Heaviside function. It is assumed that the group velocity $V_{\rm gr}$ of laser pulse propagation on the plasma plateau corresponds to the gamma factor $\gamma_{\rm ph} = 1/\sqrt{1 - V_{\rm gr}^2/c^2} = 7$ and the plasma density at the plateau corresponds to the relatio $k_0/k_{\rm p} = \gamma_{\rm ph}$, where $k_0 = 2\pi/\lambda_0$, $\lambda_0 = 1$ μm is the laser radiation wavelength. A laser pulse with such characteristics, when interacting with a plasma of a given density, excites longitudinal oscillations of electrons with an energy $W_{\rm os} = 7mc^2$ at the plateau of its profile, that is, the propagation of a wake wave in the plasma occurs on the verge of its breaking.

Figure 1 shows for different thicknesses of the transition layer the results of calculations of the relative position of the laser pulse $|e|A/mc^2$, wake potential $|e|\varphi/mc^2$, force $F_z = -|e|E_z/mc\omega_{\rm p}$, acting on electrons in the wakefield, and electron macroparticles at the time, when an electron with an oscillation centre $z_0 = A_{\rm m}$ in the process of its motion comes to the point of intersection of its trajectory with the trajectory of a neighbouring electron, which corresponds to the beginning of the process of breaking the wake wave.

Since for the chosen parameters of the laser pulse, the total energy of longitudinal oscillations of the electron is equal to the threshold energy of the wake wave breaking, then under such conditions the breaking itself does not occur and an electron bunch of an infinitely small charge is generated. In the calculations, it was assumed that the main volume ($z \geqslant 0$) of a plasma with a constant density n_0 is separated from the vacuum by a transition layer, in which the plasma density changes as $n(z) = n_0\exp[-z^2/(\pi D^2)]$, where D is the characteristic thickness of the transition layer. It follows from Fig. 1 that in the region of the plateau the nature of the wake wave and the distribution of electrons are similar, while in the region of the transition layer they are very different. It can be seen that in the 'thick' transition layer at $D \gg \lambda_{\rm p}$ (see Fig. 1b), when a laser pulse penetrates into a transparent semi-infinite plasma, a wake wave is formed at the trailing edge of the laser pulse, the characteristics of which (amplitude, oscillation period) smoothly change along the transition layer. At the same time, under conditions when the thickness of the transition layer is comparable to the size of the wake wave, a sharp change in its parameters is observed. This is because under the action of a laser pulse, the motion of electrons that were initially near the edge of the plasma density, both in the transition layer and on the density plateau, substantially depends on the thickness of the transition layer. For this reason, it should be expected that in the case of a higher-power laser pulse acting on the plasma, which transfers energy to the plasma electrons exceeding the threshold value, the nature of the wake wave breaking will depend on the thickness of the transition layer, since with an increase in the electron oscillation amplitude, more intense probing of the transition plasma layer by electrons from the plateau will occur.

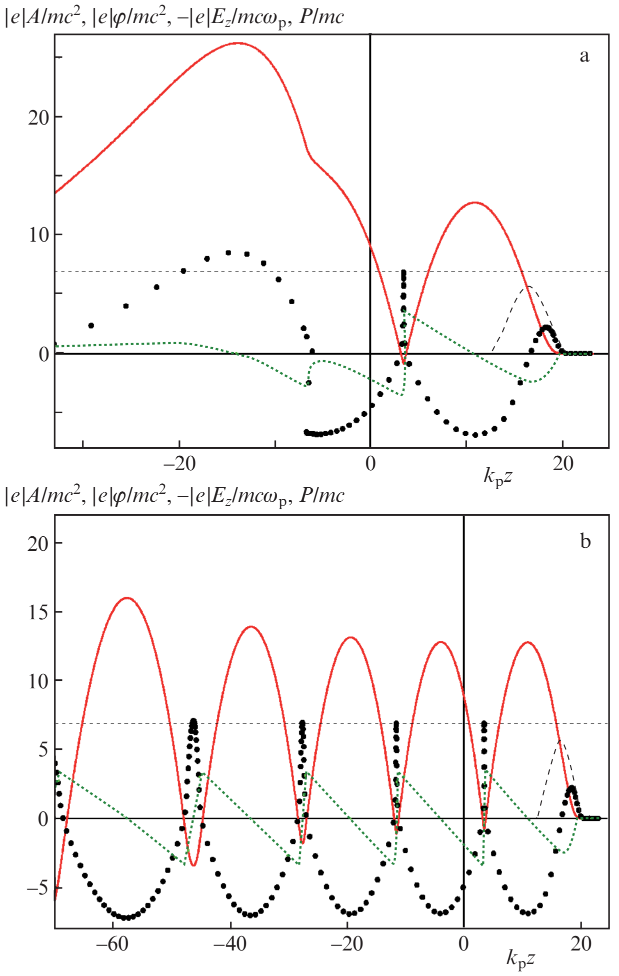

Figure 1. Relative position of the laser pulse $|e|A/mc^2$ (dashed line), wake potential $|e|\varphi/mc^2$ (solid curve), force $F_z = -|e|E_z/mc\omega_{\rm p}$, acting on electrons in the wakefield (dotted curve), and electron macroparticles (circles) at the moment the wake wave begins to break, the transition layer thickness being D = (a) 8 and (b) 80 μm. The horizontal dashed line marks the momentum of an electron macroparticle corresponding to the total oscillation energy $W_{\rm os}$.

5. Breaking of the wake wave at the above-threshold electron oscillation energy

Let us consider the situation when a laser pulse penetrating into the plasma transfers to plasma electrons an energy greater than its threshold value (8) determined by the group velocity of the pulse. In this case, the wake wave excited by the laser pulse will inevitably break. It is known [8, 9] that in the case of a sharp plasma boundary, the electron, from which the wake wave break begins in the above-threshold process, still has its oscillation centre at the same point $z_0 = A_{\rm m}$, as at the threshold value of the electron oscillation energy, and the breaking point of the wave is determined by the relation

$$z_{\rm br} = A_m - k_{\rm p}^{-1}\sqrt{2(W_{\rm os}/mc^2 - \gamma_{\rm ph})}\,. \qquad (9)$$

It is clear that in the case of a small excess above the threshold, $W_{\rm os}/mc^2 - \gamma_{\rm ph} \ll \gamma_{\rm ph}$, the breaking point $z_{\rm br}$ is located in the plasma volume, and not in the vacuum region, where the near-boundary electrons fly out in the case of a sharp boundary.

In a plasma with a blurred boundary, with the total energy of electrons slightly above the threshold, the centre of oscillation of the electron, from which the breaking process begins, as well as the point of the wake wave breaking, will be slightly displaced from the corresponding points in plasma with a sharp boundary. In other words, in the case of a blurred plasma boundary with not too high electron oscillation energy $W_{\rm os}$, one should expect that the wake wave breaking would occur in the region of the plasma density plateau. Then we can write the equation for the trajectories of electrons (3) in a simpler form:

$$ct = \frac{cz_0}{V_{\rm gr}} + I_1(z, z_0, z_0) + cT_{\rm lft}(z_0), \qquad (10)$$

where

$$I_1(z_1, z_2, z_0) = \int_{z_2}^{z_1} {\rm d}z' / \sqrt{1 - m^2c^4/[W_{\rm os} - 2\pi e^2 n_0^2 (z' - z_0)]^2}. \qquad (11)$$

Relations (10), (11), similarly to relations (3), (4), describe the trajectories of electrons forming a wake wave in that part of it where electrons move from the leftmost point of their trajectory to the rightmost one. In this case, only the first period of oscillations is considered. The last term in Eqn (10) takes into account that in the course of its motion, an electron can enter the transition layer. It should be noted that the passage of an electron through the transition layer changes the phase shift between the oscillations of individual electrons, which is a significant factor affecting the nature of the intersection of electron trajectories. In this case, the similarity of the electron trajectories in the region of the plasma density plateau is restored, albeit with a different phase shift between their oscillations. This circumstance makes it possible to simplify the mathematical description of the physical phenomenon under consideration.

Applying the condition of intersection of electron trajectories ${\rm d}Z/{\rm d}z_0 = 0$ to the trajectories described by Eqns (10), (11), we obtain for an arbitrary electron from the plasma density plateau the relation

$$\frac{c}{u(z_{\rm cr}, z_0)} = \frac{1}{\beta} + c\frac{\partial T_{\rm lft}(z_0, W_{\rm os})}{\partial z_0}, \qquad (12)$$

where $u(z_{\rm cr}, z_0)$ is the velocity of an electron with an oscillation centre at point z_0 at the moment of its trajectory intersection with the trajectory of a neighboring electron at point $z_{\rm cr}$.

On the other hand, expression (10) can be used to calculate the phase velocity of the wake wave. However, since the trajectories of electrons described by relations (10) and (11) are not only nonharmonic, but also generally even nonsimilar due to the electrons entering the transition layer, it is first necessary to generalise the usual definition of the concept of the phase of a wake wave to the case of such trajectories. Below, by the phase of the wake wave at any point z in space at any fixed time moment we mean the deviation Δ from the centre of oscillations z_0 of the electron that is at the point z at a given time. To find out the phase velocity of a wake wave, which has a complex irregular character, it is necessary to trace the velocity of its phase movement for a preselected and fixed value of $\Delta = z_0 - z$. Replacing in Eqns (10), (11) $z_0 = z_\Delta + \Delta$ and calculating the derivative ${\rm d}z_\Delta/{\rm d}t$, we can determine the phase velocity of propagation of any phase of an irregular wake wave:

$$\frac{{\rm d}z_\Delta}{{\rm d}t} = c\left[\frac{1}{\beta} + c\frac{\partial T_{\rm lft}(z_\Delta + \Delta, W_{\rm os})}{\partial z_0}\right]^{-1}. \qquad (13)$$

Comparing Eqns (12) and (13), we see that at the moment of intersection of the trajectories of electrons, their velocity is equal to the velocity of the wake wave phase, determined by the above method. It also follows from Eqn (13) that the irregular wake wave, which is formed by electrons that have passed through the transition layer, has a phase velocity that depends on the spatial coordinate, and each phase of the irregular wake wave propagates with its own phase velocity. Thus, the irregular wave also differs by this from the usual regular wake wave for a laser pulse propagating in a homogeneous plasma, in which the phase velocity is the same for all its oscillation phases. The size of the boundary region on the plasma density plateau, in which the wake wave is irregular, is determined by the dependence $\partial T_{\rm lft}(z_\Delta + \Delta, W_{\rm os})/\partial z_0$ and, therefore, is equal to the amplitude of electron oscillations $A_{\rm m}$.

The complex wave motion of the plasma, which is realised near its boundary, turns into a regime of wave breaking and generating an electron bunch, when at a certain point in space at a certain moment in time it turns out that the velocity of an electron located at this point coincides with the phase velocity of the wake wave. The point in space $z_{\rm br}$, from which the process of the wake wave breaking begins, is the point where condition (12) occurs earlier.

According to relation (10), each pair of values z_0 and $z_{\rm cr}$ corresponds to the time $t_{\rm cr}$ of the intersection of the trajectories. Calculating the derivative ${\rm d}z_{\rm cr}/{\rm d}z_0$ with Eqn (12) taken into account, we obtain

$$\frac{{\rm d}t_{\rm cr}}{{\rm d}z_0} = \frac{{\rm d}z_{\rm cr}}{{\rm d}z_0}\frac{1}{u(z_{\rm cr}, z_0)}. \qquad (14)$$

From Eqn (14) it follows that the condition ${\rm d}z_{\rm cr}/{\rm d}z_0 = 0$ determines the centre of oscillation $z_{0\,\rm f}$ of the electron from which the process of the wake wave breaking begins. Then, from the relation

$$z_{\rm cr} = z_0 - k_{\rm p}^{-1} \times \sqrt{2\left\{\frac{W_{\rm os}}{mc^2} - 1\Big/\sqrt{1 - 1\Big/\left[\frac{1}{\beta} + c\frac{\partial T_{\rm lft}(z_0, W_{\rm os})}{\partial z_0}\right]^2}\right\}} \qquad (15)$$

we find the spatial position of the breaking point $z_{\rm br}$ and from relation (10) we find the breaking time $t_{\rm br}$.

However, a practical solution to the problem of exact analytical determination of the minimum of function (15) is not possible. Therefore, we use an approximate representation of the function $T_{\rm lft}(z_0, W_{\rm os})$ for points $z_0 \leqslant A_{\rm m}$ under the condition $A_{\rm m} - z_0 \ll A_{\rm m}$:

$$T_{\text{lft}}(z_0) = T_{\text{h}} + \omega_{\text{p}}^{-1}\frac{32\sqrt{2}}{105\pi}\frac{(A_{\text{m}} - z_0)^{7/2}}{A_{\text{m}}^{3/2}D^2}. \qquad (16)$$

The study of Eqn (16) shows that when the relation $W_{\text{os}}/(mc^2) - \gamma_{\text{ph}} \gg 0.06 k_{\text{p}}^{14} A_m^6 D^8 / \gamma_{\text{ph}}^{12}$ is satisfied, the minimum of function (16) is determined by the expression

$$z_{0\,\text{f}} = A_{\text{m}}\left\{1 - \left(\frac{3\pi}{8}\right)^{2/3}\frac{[W_{\text{os}}/(mc^2) - \gamma_{\text{ph}}]^{1/3}(k_{\text{p}}D)^{4/3}}{\gamma_{\text{ph}}^2}\right\}. \qquad (17)$$

It follows from Eqn (17) that in the above-threshold process in the case of a blurred plasma boundary the wake wave breaking occurs in such a way that the centre of electron oscillations, from which the breaking process begins, shifts closer to the edge of the plasma density. The more shallow the transition layer, the greater this displacement. The displacement also increases with increasing overthreshold level of the breaking process. With a large overthreshold and with a very shallow transition layer, the electron, from which the process of breaking the wake wave begins, can be displaced into the transition layer. This is confirmed by simulation, but under such conditions the approximate formula (17) becomes inapplicable.

The reason for the shift of the breakiing point into the transition layer for a gently sloping plasma profile is clear from Fig. 1b, which demonstrates that in a very flat transition layer, the wake wave differs insignificantly in its parameters from the wave formed on the plasma density plateau. And although Eqn (17) is inapplicable in this region, the analytical method used in this study can also be applied to the transition layer if the phase velocity of the wake wave changes weakly.

Figure 2 shows the plots of the change in the phase velocity of the wake wave along the spatial coordinate for the value of the phase at which the wake potential has a minimum for different thicknesses of the transition layer. The figure shows that the thinner the transition layer, the higher the phase velocity of the wake wave in it. The values of the phase velocity in the thin transition layer are much higher than the speed of light; therefore, the breaking of the wake wave in such a transition layer becomes impossible. And vice versa, if the degree of blurring of the plasma boundary is very high, then the phase velocity of the wake wave in the layer can be less than the light velocity, and if the breaking threshold is sufficiently exceeded, the destruction of the wake wave with the generation of an electron bunch can begin in the transition layer, and not in the region of the plasma density plateau.

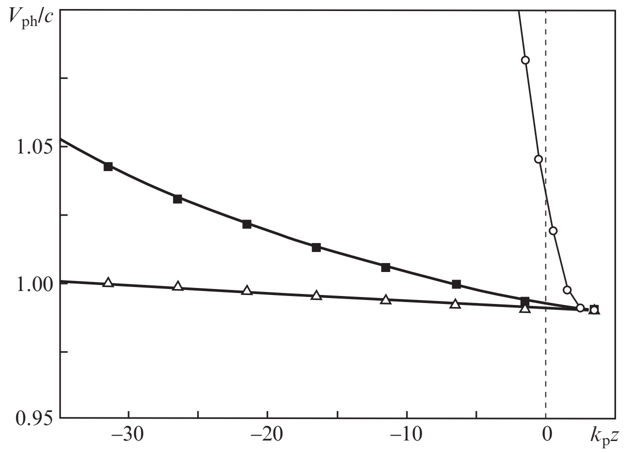

Figure 2. Dependence of the phase velocity of the wake wave on the coordinate along the plasma profile for a transition layer thickness of (○) 8, (■) 40, and (△) 80 μm. The vertical dashed line shows the position of the density edge of the transition plasma layer.

The presented results, obtained for one-dimensional geometry, in a real situation of a focused laser pulse with a characteristic transverse size $r \sim \sigma_{\text{las}}$ always relate to the initial stage of electron trapping in its wake wave. In Ref. [6], it was shown that at the first stage, by means of longitudinal self-injection into the bunch formed for acceleration, paraxial plasma electrons ($r \ll \sigma_{\text{las}}$) are trapped, which pass through the laser pulse, practically not deflecting in the transverse direction due to the smallness of the transverse component of the ponderomotive force for them. At the second stage, with some time delay, by means of transverse self-injection, electrons are trapped into the bunch formed for acceleration, which are initially located at a distance $k_{\text{p}}\sigma_{\text{las}} \gg \sqrt{a_0}$ [7] from the laser pulse axis. This occurs in the bubble regime [14–16], when the ponderomotive force displaces electrons from the propagation axis of the laser pulse and forms a cavity free of electrons in its wake. Electrons circulate around the laser pulse and bubble and reach the phase velocity of the wake near the back of the bubble.

Usually, upon a sufficiently strong focusing of the laser pulse, the charge of the trapped bunch is formed mainly due to the mechanism of transverse self-injection. However, to take advantage of the trapped and accelerated electron bunch quality that the longitudinal self-injection mechanism provides, this mechanism can be made dominant. For this purpose, as verified by numerical simulation, the laser pulse should be wide enough, $k_{\text{p}}\sigma_{\text{las}} \gg \sqrt{a_0}$ [7], and no cavity behind the laser pulse is formed. For a plasma density corresponding to $k_0/k_{\text{p}} = 7$ and a laser pulse with an amplitude $a_0 = 5.652$ and a characteristic transverse size $\sigma_{\text{las}} = 20\lambda_0$, this condition is satisfied. Then the motion of plasma electrons will be approximately one-dimensional, and the study of the process of breaking the wake wave can be carried out in one-dimensional geometry.

6. Conclusions

An analytical study of the penetration of a laser pulse of relativistic intensity into a semi-infinite rarefied plasma with a transition layer at the boundary, carried out in one-dimensional geometry, made it possible to clarify the properties of the wake wave generated by the laser pulse and the conditions for its breaking. It is shown that the wake wave generated after the laser pulse passing through the diffuse plasma boundary is irregular not only in the transition layer, but also in a certain region of the plasma density plateau near the boundary. The thickness of this region on the plasma plateau is equal to the amplitude of the oscillations of electrons, whose longitudinal oscillations are excited by a laser pulse. The phase velocity of the wake wave in this area depends on the spatial coordinate, and each phase of the oscillation has its own phase velocity.

It was found that the process of wake wave breaking has a threshold character and becomes possible if the total energy of longitudinal oscillations of electrons exceeds the gamma factor determined by the group velocity of the laser pulse on the plasma density plateau. Thus, the breaking threshold is independent of the shape of the transition layer at the plasma boundary. It is shown that at the threshold value of the electron oscillation energy, the process of wake wave breaking begins with the electron that is initially located at a distance equal to the amplitude of its oscillations from the edge of the transition layer. It was found that with an increase in the overthreshold value of the breaking process, the centre of electron oscillation, from which the breaking process begins, shifts closer to the plasma boundary. The mildler the slope of the transition layer at the plasma boundary, the greater this displacement. The excess of the electron oscillation energy over the threshold value also shifts the breaking point closer to the plasma boundary.

Acknowledgements. The reported study was funded by RFBR and ROSATOM, project number 20-21-00150.

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