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Line shape of the coherent population trapping resonance in the case of a Gaussian spatial profile of a light beam

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Abstract. **For coherent population trapping (CPT) resonance excited by a bichromatic field in a closed Λ-system, we have obtained an analytical expression for the line shape in the case of a Gaussian profile of the radiation intensity distribution, taking into account the light shift of the reference transition frequency. Due to the spatial inhomogeneity of the light shift, the line shape is asymmetric. The dependence of the shift of the resonance peak position on the light intenisty is investigated.**

Keywords: *coherent population trapping resonance, light shift, laser spectroscopy, atomic clocks and magnetometers.*

1. Introduction

Progress on the development of atomic clocks have led to numerous advances in various fields of science and technology: global navigation satellite systems, high-speed telecommunications, secure communication lines, relativistic geodesy, verification of fundamental physical theories, etc. [1–9]. There is a wide demand for passive atomic clocks of the microwave range with a gas optical cell, in which a quantum transition between the components of the hyperfine structure of the ground state of alkali metal atoms is used as a reference for frequency stabilisation. In this frequency range, atomic clocks based on the coherent population trapping (CPT) effect are of great interest [10–14]. This effect is observed as a narrow dip in the absorption/fluorescence signal (or a narrow peak in the transmission signal) when atoms are excited by a bichromatic (two-frequency) field. The condition for the CPT appearance is the equality of the difference between the frequencies of the exciting optical fields and the frequency of hyperfine splitting in the ground state of the atom. In this case, a so-called dark state is formed, in which atoms do not absorb light. The use of only optical field for clock transition spectroscopy is an important advantage of CPT clocks, since in this case there is no microwave cavity in the design, which makes it possible to achieve significant miniaturisation of the device (including chip-scale) and low power consumption [15–17].

The line shape of the spectroscopic signal and the shift of the resonance frequency are the key physical parameters that determine the metrological characteristics of quantum sensors based on CPT resonances. Knappe et al. [18] showed that in the general case, the line shape can be modelled as a sum of symmetric (Lorentzian) and antisymmetric (dispersive) line shapes. The antisymmetric contribution leads to an asymmetry of the resonance and arises under the condition that the one-photon detuning is nonzero, with the amplitudes of the light fields being unequal to each other. This asymmetry is an additional source of the error signal shift [19] and can have a significant negative effect on the stability of the frequency standard. In addition, the line shape depends on the spatial profile of the radiation intensity. For example, Taichenachev et al. [20] considered the influence of a Gaussian transverse field profile on the resonant line shape. They demonstrated that the dependences of the resonance width and amplitude on the intensity differ significantly from the situation when the radiation intensity is uniform over the beam cross section. However, the light (ac Stark) shift, which is also nonuniform over the light beam cross section, was not taken into account in [20].

In this work, we study the effect of the Gaussian profile of the laser beam on the line shape taking into account the light shift, which depends on the local light intensity. It is shown that the spatial inhomogeneity of the light shift results in the line shape asymmetry. In addition, the Gaussian profile leads to a substantially nonlinear character of the dependence of the resonance peak shift on the light intensity.

2. Theoretical model

As a theoretical model of an atomic medium, we consider a closed three-level Λ-system (Fig. 1), interacting with a bichromatic (two-frequency) field:

$$E(t) = E_1 e^{-i\omega_1 t} + E_2 e^{-i\omega_2 t} + \text{c.c.} \tag{1}$$

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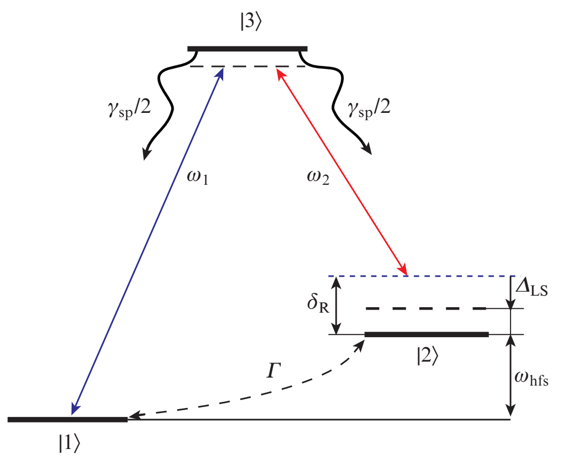

Figure 1. Scheme of a three-level Λ-system.

The CPT resonance is excited under the condition that the frequency difference $\omega_1 - \omega_2$ is scanned near the frequency $\omega_{\rm hfs}$ of the transition between the lower states of the Λ-system. The temporal dynamics of this quantum system will be described using the atomic density matrix formalism

$$\hat{\rho}(t) = \sum_{m,n} |m\rangle\, \rho_{mn}(t)\, \langle n|, \tag{2}$$

where $m,n = \{1,2,3\}$. In the rotating wave approximation, the equations for the components of the density matrix $\rho_{mn}(t)$ have the form:

$$\begin{aligned}
&\partial_t\rho_{11} = -\Gamma(\rho_{11} - 1/2) + (\gamma_{\rm sp}/2)\rho_{33} + {\rm i}(\Omega_1^*\rho_{31} - \Omega_1\rho_{13}),\\
&\partial_t\rho_{12} = [-\Gamma - {\rm i}(\delta_{\rm R} - \Delta_{\rm LS})]\rho_{12} - {\rm i}(\Omega_2\rho_{13} - \Omega_1^*\rho_{32}),\\
&\partial_t\rho_{21} = [-\Gamma + {\rm i}(\delta_{\rm R} - \Delta_{\rm LS})]\rho_{21} + {\rm i}(\Omega_2^*\rho_{31} - \Omega_1\rho_{23}),\\
&\partial_t\rho_{22} = -\Gamma(\rho_{22} - 1/2) + (\gamma_{\rm sp}/2)\rho_{33} + {\rm i}(\Omega_2^*\rho_{32} - \Omega_2\rho_{23}),\\
&\partial_t\rho_{13} = (-\gamma_{\rm opt} - {\rm i}\delta_1)\rho_{13} - {\rm i}\Omega_1^*(\rho_{11} - \rho_{33}) - {\rm i}\Omega_2^*\rho_{12},\\
&\partial_t\rho_{31} = (-\gamma_{\rm opt} + {\rm i}\delta_1)\rho_{31} + {\rm i}\Omega_1(\rho_{11} - \rho_{33}) + {\rm i}\Omega_2\rho_{21},\\
&\partial_t\rho_{23} = (-\gamma_{\rm opt} - {\rm i}\delta_2)\rho_{23} - {\rm i}\Omega_2^*(\rho_{22} - \rho_{33}) - {\rm i}\Omega_1^*\rho_{21},\\
&\partial_t\rho_{32} = (-\gamma_{\rm opt} + {\rm i}\delta_2)\rho_{32} + {\rm i}\Omega_2(\rho_{22} - \rho_{33}) + {\rm i}\Omega_1\rho_{12},\\
&\partial_t\rho_{33} = -(\Gamma + \gamma_{\rm sp})\rho_{33} + {\rm i}(\Omega_1\rho_{13} - \Omega_1^*\rho_{31})\\
&\qquad + {\rm i}(\Omega_2\rho_{23} - \Omega_2^*\rho_{32}),
\end{aligned} \tag{3}$$

where $\Omega_1 = d_{31}E_1/\hbar$ and $\Omega_2 = d_{32}E_2/\hbar$ are the Rabi frequencies for the $|1\rangle \leftrightarrow |3\rangle$ and $|2\rangle \leftrightarrow |3\rangle$ transitions, respectively (d_{31} and d_{32} are the matrix elements of the operator of the electric dipole moment of the transition); $\delta_1 = \omega_1 - \omega_{31}$ and $\delta_2 = \omega_2 - \omega_{32}$ are single-photon detunings of laser fields; $\delta_{\rm R} = \omega_1 - \omega_2 - \omega_{\rm hfs}$ is the two-photon (Raman) detuning for unperturbed transition $|1\rangle \leftrightarrow |2\rangle$; $\Delta_{\rm LS}$ is the light shift of the clock transition frequency; $\gamma_{\rm opt}$ is the decay rate of optical coherences (due to spontaneous decay processes, collisions with buffer gas atoms, etc.); $\gamma_{\rm sp}$ is the rate of spontaneous decay of the excited state; and the constant Γ models the relaxation of atoms (for example, due to time-of-flight effects) in the absence of a field to an equilibrium distribution over states $|1\rangle$ and $|2\rangle$.

Let us determine the spatial dependences of the transverse profile f on the radius r in the Rabi frequencies and light shift:

$$\Omega_1^2(r) = \Omega_{10}^2 f(r),\ \ \Omega_2^2(r) = \Omega_{20}^2 f(r),\ \ \Delta_{\rm LS}(r) = \Delta_0 f(r), \tag{4}$$

where Ω_{10}, Ω_{20}, and Δ_0 are the Rabi frequencies and the light shift on the axis (i.e., at $r = 0$) of the light beam, respectively. In the case of a Gaussian light beam profile, the function $f(r)$ has the form

$$f(r) = {\rm e}^{-r^2/r_0^2}, \tag{5}$$

where the radius r_0 determines the characteristic transverse size of the beam. In an optically thin medium, the absorption of light can be assumed to be proportional to the integral value of the excited state population:

$$\langle\rho_{33}\rangle = \int_0^\infty \int_0^{2\pi} \rho_{33}(r)\, r\, {\rm d}\varphi\, {\rm d}r, \tag{6}$$

which we will analyse below.

3. Resonance line shape

The stationary solution of the system of equations (3) for the local population of the excited state $\rho_{33}(r)$ under the conditions of single-photon resonance (i.e., at $|\delta_{1,2}| \ll \gamma_{\rm opt}$) can be represented in the form of a Lorentzian:

$$\rho_{33}(r) = B(r) + \frac{A(r)\gamma_{\rm CPT}^2(r)}{[\delta_{\rm R} - \Delta_{\rm LS}(r)]^2 + \gamma_{\rm CPT}^2(r)}. \tag{7}$$

In the small saturation approximation ($\Omega_{1,2}^2 \ll \gamma_{\rm sp}\gamma_{\rm opt}$), the parameters $A(r)$, $B(r)$, and $\gamma_{\rm CPT}(r)$ in formula (7) take the form

$$A(r) = -\frac{4\Gamma\beta W_0^2 f^2(r)}{\gamma_{\rm sp}[1 + W_0 f(r)]}, \tag{8}$$

$$B(r) = \frac{\Gamma W_0 f(r)[1 + 4\beta W_0 f(r)]}{\gamma_{\rm sp}[1 + W_0 f(r)]}, \tag{9}$$

$$\gamma_{\rm CPT}(r) = \Gamma[1 + W_0 f(r)], \tag{10}$$

where

$$W_0 = \frac{\Omega_{10}^2 + \Omega_{20}^2}{\Gamma\gamma_{\rm opt}},\ \ \beta = \frac{\Omega_{10}^2/\Omega_{20}^2}{(1 + \Omega_{10}^2/\Omega_{20}^2)^2}. \tag{11}$$

Substituting (7)–(10) into formula (6), we obtain an analytical expression for the shape of the CPT resonance line in the case of a Gaussian light beam profile (5):

$$\begin{aligned}
\langle\rho_{33}\rangle = \pi r_0^2 \frac{\Gamma}{\gamma_{\rm sp}}&[4\beta W_0 + (1 - 4\beta)\ln(1 + W_0)\\
&- 4\beta W_0^2 F(\delta_{\rm R})],
\end{aligned} \tag{12}$$

where the dimensionless function $F(\delta_{\rm R})$ describing the resonance shape is defined as

$$F(\bar{\delta}_R) = \frac{W_0}{W_0^2 + \bar{\Delta}_0^2} - \frac{(W_0^2 - \bar{\Delta}_0^2) - 2W_0\bar{\Delta}_0\bar{\delta}_R}{2(W_0^2 + \bar{\Delta}_0^2)^2}\ln\frac{(1 + W_0)^2 + (\bar{\delta}_R - \bar{\Delta}_0)^2}{1 + \bar{\delta}_R^2} - \frac{2W_0\bar{\Delta}_0 + (W_0^2 - \bar{\Delta}_0^2)\bar{\delta}_R}{(W_0^2 + \bar{\Delta}_0^2)^2} \times\left[\arctan\frac{W_0(1 + W_0) + \bar{\Delta}_0^2 - \bar{\Delta}_0\bar{\delta}_R}{W_0\bar{\delta}_R + \bar{\Delta}_0} - \arctan\frac{W_0 - \bar{\Delta}_0\bar{\delta}_R}{W_0\bar{\delta}_R + \bar{\Delta}_0}\right]. \quad (13)$$

Here, for the sake of brevity, we use dimensionless quantities

$$\bar{\delta}_R = \delta_R/\Gamma,\ \bar{\Delta}_0 = \Delta_0/\Gamma. \quad (14)$$

As can be seen from Fig. 2, the spatial inhomogeneity of the light shift is a source of the asymmetry of the integral line shape of CPT resonance.

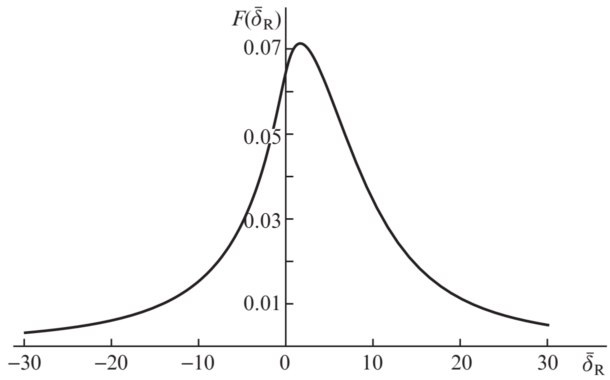

Figure 2. CPT resonance line shape $F(\bar{\delta}_R)$ in the case of a Gaussian light beam profile taking into account a spatially inhomogeneous light shift. Numerical parameters of the model: $W_0 = 10$, $\bar{\Delta}_0 = 0.5W_0$.

As a rule, the light shift under real conditions is small in comparison with the resonance width. In this case, from formula (13) we obtain the approximate expression for the shift of the resonance peak:

$$\delta_{top} \approx -\frac{\Gamma[(1 + W_0)^2 + \bar{\Delta}_0^2]^2}{[(1 + W_0)^2 - \bar{\Delta}_0^2](W_0^2 + \bar{\Delta}_0^2)^2} \times\left[\frac{\bar{\Delta}_0(1 + 2W_0)(W_0^2 + \bar{\Delta}_0^2)}{(1 + W_0)^2 + \bar{\Delta}_0^2} + (W_0^2 - \bar{\Delta}_0^2)\arctan\frac{\bar{\Delta}_0}{1 + W_0} + \bar{\Delta}_0 W_0 \ln\frac{1}{(1 + W_0)^2 + \bar{\Delta}_0^2}\right]. \quad (15)$$

Figure 3 shows the dependence of the shift of the resonance peak on the light intensity. One can see that the Gaussian profile of the light shift leads to a substantially nonlinear character of this dependence.

Thus, we have considered the influence of the Gaussian profile of the laser beam on the shape of the CPT resonance line taking into account the spatially inhomogeneous light shift. In the case of a closed Λ-system, we have obtained an analytical expression that describes the resonance line shape, which has an asymmetric form. We have shown that the dependence of the shift of the resonance peak on the light intensity is essentially nonlinear.

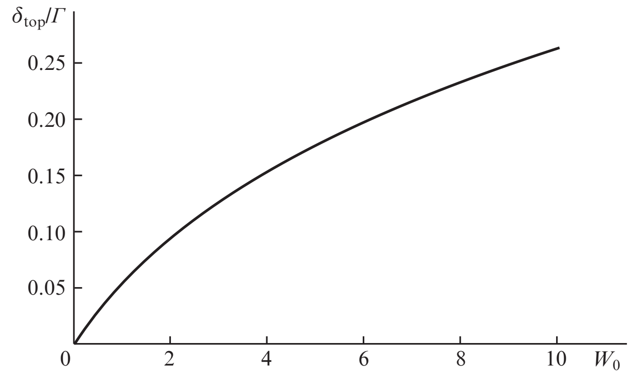

Figure 3. Shift of the CPT resonance peak vs. intensity on the light beam axis at $\bar{\Delta}_0 = 0.1W_0$.

It should be noted that in addition to the Gaussian transverse intensity profile, the asymmetry of the CPT resonance will also be influenced by the absorption of light, leading to a spatial inhomogeneity of the light shift in the longitudinal direction. However, this issue requires separate consideration.

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