Magneto-optical trap for ⁶ Li atoms formed by elliptically polarised light waves

R.Ya. Il'enkov, A.A. Kirpichnikova, O.N. Prudnikov

Abstract. **We report a study of a model of a magneto-optical trap (MOT)** for atoms ⁶Li in the field with $\varepsilon - \theta - \bar{\varepsilon}$ configuration pro**duced by the counterpropagating elliptically polarised waves, which** are in resonance with the optical transition ${}^{2}S_{1/2} \rightarrow {}^{2}P_{3/2}$ (λ = **670.977 nm). The model takes into account hyperfine splitting of levels inside the natural linewidth of the optical transition. In contrast to a conventional MOT formed by counterpropagating circu**larly polarised waves $(\sigma_{+} - \sigma_{-})$ configuration), the suggested MOT **may provide a deeper sub-Doppler cooling of 6Li atoms.**

Keywords: magneto-optical trap, sub-Doppler cooling, laser cooling, elliptic polarisation.

1. Introduction

Presently, laser cooling of atoms is widely used in modern research for developing quantum sensors based on atomic interferometers (gyroscopes, gravimeters, and accelerometers); obtaining Bose condensates [1, 2] and Fermi gases [3]; for developing quantum computers [4]; designing modern optical frequency standards $[5-7]$, widely used both in fundamental studies for measuring a geopotential, fundamental constants, verifying the relativity theory and in applied problems, for example, fabricating modern navigation systems [8]. This requires the development of efficient methods for deep laser cooling of atoms.

Despite of the fact that the theory of laser cooling is well developed [9, 10], the problem of choosing the optimal parameters and configuration of a light field in laser cooling is still open for particular conditions of experimental realisation. In the frameworks of the semi-classical theory $[9-14]$, it is noted that a temperature of sub-Doppler laser cooling in low-intensity fields may be below the Doppler limit and amount to several recoil energy. However, a full-scale analysis of atom kinetics performed in [15, 16] has shown that the known process of sub-Doppler cooling only occurs for atoms possessing the limiting small recoil parameters $\varepsilon_r = \omega_r / \gamma \ll 10^{-3}$, that is,

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the ratio of the recoil energy $\hbar \omega_r = \hbar k^2 / 2M$, acquired by an atom of mass *M* in the result of single act of photon absorption or emission with a wavevector k , to the natural linewidth γ of the optical transition involved in the laser cooling. In this case, even in the conditions of the kinetic semi-classical approximation, atomic sub-Doppler temperatures may be unreachable $[15-18]$ and at high values of ε_r (0.01 – 0.1) effects of sub-Doppler cooling become less efficient [15, 16] especially in the $\sigma_+ - \sigma_-$ field configuration, which is conventionally used in magneto-optical traps (MOTs).

It is also important that the equilibrium distribution of cooled atoms in the light field is not Gaussian; hence, it cannot be described in terms of temperature [19]. However, it can be described in the frameworks of the two-temperature distribution of atoms possessing a cold and hot fractions [20].

Among alkali atoms, the lithium atom is rather promising for studying quantum effects arising under extremely low temperatures since it is the lightest in this group of elements. Lithium is also ideal for working with Bose [21] and Fermi condensates [22] of atoms, because it has stable isotopes with both integer and half-integer nuclear spins: 6Li ($I = 1$; 7.5%) and ⁷Li ($I = 3/2$; 92.5%). For obtaining a considerable density of lithium atoms in traps, the cooling starts from higher temperatures (due to a small atomic mass) and the process of atom trapping and cooling in a MOT is difficult [23]. Note some works on the sympathetic method for cooling Li atoms in a MOT, which imposes certain limitations on the following employment of cold atoms due to difficulties of the total extraction of buffer atoms from the trap [24 – 26].

In the frameworks of a theoretical analysis, note that lithium belongs to atoms with an insufficiently small $({\sim}10^{-2})$ value of parameter ε_r , which, according to [15, 16], hinders obtaining sub-Doppler temperatures. In addition, the natural linewidth $g \sim 5.8$ MHz [27] is greater than the hyperfine splitting of the excited state, which prevents the employment of classical theories and models in the analysis.

An analysis of laser cooling of lithium atoms requires a more complicated model than those considered in standard classical theories, which will give a possibility to determine more optimal configurations and parameters of the light fields for obtaining deep laser cooling.

For obtaining sub-Doppler laser cooling of ⁶Li atoms, the present work suggests the employment of the MOT formed by elliptically polarised light waves.

2. Hyperfine structure of lithium atoms

For cooling 6Li atoms, various optical transitions corresponding to lines D1 and D2 (Fig. 1) can be used. No closed transition can be selected in the hyperfine structure of atoms; hence, an additional pump field is needed, which will return atoms to a closed cooling cycle. Issuing from preliminary estimates, for thorough investigation we have selected the transition ${}^{2}S_{1/2} \rightarrow {}^{2}P_{3/2}$ (Fig. 1). The additional pump field resonant to the transition ${}^2S_{1/2} \rightarrow {}^2P_{1/2}$ is used to return atoms to the cycle of interaction with a cooling field.

Figure 1. (Colour online) Schematic of a hyperfine structure of the ⁶Li atom [27]. The green line shows the cooling field, and the blue line shows the pump field.

Parameters of D1 and D2 lines are given in Table 1 [27]. In contrast to the classical two-level model of atoms with the levels degenerated with respect to angular momentum projections in the ground and excited states, in the considered model we take into account all the hyperfine structure sublevels, which interact with the field resonant to the transition ${}^{2}S_{1/2} \rightarrow {}^{2}P_{3/2}$.

We will study the limits of laser cooling by using a semiclassical approach based on a solution of the Fokker –Planck (FP) equation $[28-31]$. The kinetic coefficients in this equation are the force acting on atom in a light field and the diffusion coefficient arising in the result of force fluctuations in processes of field photon emission and absorption.

3. Problem statement

Consider a one-dimensional problem for laser cooling of atoms in the light field formed by a pair of waves of equal intensity counterpropagating along *z* axis:

$$
\mathbf{E}(z,t) = E_0(e_1 \exp(ikz) + e_2 \exp(-ikz)) \exp(-i\omega t) + \text{c.c. (1)}
$$

Here, E_0 is the complex amplitude of the light field; and e_1 and $e₂$ are the polarisation vectors of the counterpropagating waves, which determine a spatial configuration of the field. In the case, where vectors e_1 and e_2 correspond to orthogonal linear polarisations, a so-called lin \perp lin configuration of a field is formed. When the vectors correspond to circular polarisations, a σ_{+} – σ_{-} configuration is formed, in which a sub-Doppler cooling of atoms may occur [13].

In the semi-classical approximation $[9-14]$, a one-dimensional problem of laser cooling is described by the FP equation for the atomic distribution function in a phase space $W(z, p, t)$. For further kinetic analysis, we choose the following form for this equation:

$$
\frac{\partial}{\partial t}W + \frac{p}{M}\frac{\partial}{\partial z}W = -\frac{\partial}{\partial p}(FW) + \frac{\partial^2}{\partial p^2}(D_{sp}W)
$$

$$
+ \frac{\partial}{\partial p}\left(D_{\text{ind}}\frac{\partial}{\partial p}W\right).
$$
(2)

This choice is related to the fact that for atoms with an insufficiently small recoil parameter ε_r , such as lithium atoms, a solution of the FP equation written in form (2) leads to results, which are very close to those obtained numerically with quantum approaches [15, 16].

The kinetic coefficients of the FP equation are the force *F* acting on an atom in a light field and the diffusion coefficient *D*, which includes contributions from spontaneous $(D_{\rm SD})$ and induced (D_{ind}) diffusions. The kinetic coefficients can be directly obtained by reducing the quantum kinetic equation for the density matrix of atoms

$$
\frac{\partial}{\partial t}\hat{\rho} = -\frac{i}{\hbar}[\hat{H}, \hat{\rho}] - \hat{\Gamma}\{\hat{\rho}\}.
$$
 (3)

Here, \hat{H} is the Hamiltonian of the system in question; and $\hat{\Gamma}$ { $\hat{\rho}$ } is the operator of spontaneous relaxation, which describes variations of internal and external translational degrees of freedom for the density matrix under spontaneous emission of field photons (see, for example, [9, 14, 31]). In view of employing a more complicated model, which takes into account several hyperfine components of the excited state level, the expressions for the force and diffusion coefficient in the considered model of the lithium atom may differ from those considered earlier for simple models and can be obtained from the approaches presented in [14, 29, 32, 33].

4. Limits of laser cooling of 6Li. Comparison of 'simplified' and 'more complete' models

Let us analyse the kinetics of lithium atoms in a light field on a basis of a simplified model. It is a degenerated two-level

Table 1. Parameters of D1 and D2 lines of the ⁶ Li atom [27].

Line				Parameters		
	Wavelength λ /nm	Wavenumber $\frac{k}{2\pi}$ / cm^{-1}	Frequency v /THz	Lifetime τ /ns	Natural linewidth γ /s ⁻¹ (MHz)	Rate of atomic relaxation $v_{rel}/\text{cm s}^{-1}$
D ₁	670.992421	14903.298	446.789634	27.102	36.898×10^6 (5.8724)	9.886554
D ₂	670.977338	14903.633	446.799677	27.102	36.898×10^6 (5.8724)	9.886776

model with the total angular momentum $F_g = 3/2$ in the ground state and $F_e = 5/2$ in the excited state (Fig. 2a). We will compare this analysis with the kinetics based on a more complete model, which takes into account an interaction of a laser field with several hyperfine splitting levels of excited state $2^{2}P_{3/2}$ ($F_{e1} = 5/2$, $F_{e2} = 3/2$, $F_{e3} = 1/2$), which fit into the resonance interaction profile of width γ (Fig. 2b).

Figure 2. Part of the energy level diagram of the ⁶Li atom: (a) simplified and (b) more complete models; Γ is the reciprocal lifetime for the atomic excited state.

The comparative analysis performed in the frameworks of the simplified and more complete models (Fig. 2) reveals an influence of the complicated level structure of the excited state. Namely, several hyperfine components inside the interaction line profile determined by a natural width γ affect the reachable limits of lithium atom laser cooling as compared to the limits found from classical two-level models where the levels of ground and excited states are degenerated with respect to the angular momentum projection. In our analysis, we also neglect the recoil effect of the pump field (the corresponding initiated transitions are marked blue in Fig. 1), aiming mainly at the search for optimal configurations and parameters of the cooling field for deep laser cooling. The recoil induced by the pump field in the exact resonance of the transition ${}^{2}S_{1/2}$ (*F* = 1/2) $\rightarrow {}^{2}P_{1/2}$ (*F* = 1/2) leads to an additional heating of atoms, which can be minimised by varying the pump field intensity. In addition, by using the pump field configuration

in the form of a standing wave, in the general case with an inhomogeneous elliptic polarisation, one can obtain additional cooling of lithium atoms. However, because the study of these problems is beyond the scope of this work, we will mainly focus on the laser cooling by a 'cooling' field, which is resonant to the transition ${}^{2}S_{1/2}$ (*F* = 3/2) $\rightarrow {}^{2}P_{3/2}$ (*F* = 1/2, $F = 3/2$, $F = 5/2$, and on the influence of atoms with a complicated structure of excited state ${}^{2}P_{3/2}$ on the kinetics.

One more important approximation of the considered model is the employment of the single-particle approximation, where interaction effects between radiation and surrounding lithium atoms are neglected. The study of MOT operation regimes with a low density *n* of atoms in a trap, for which the relationship $n\lambda^3$ < 1 holds, so that these kinetic effects can be neglected, is of particular interest and may reveal reachable limits of atom laser cooling in a MOT. In this case, an increase in the atomic density in the MOT due to various effects related to atom interaction with a radiation of ambient cold atoms and to absorption of emission in optically dense medium, leads to an elevation of the atomic cloud temperature with the growth of cloud density [34]. Since the aim of our analysis is a comparison of various field configurations and determination of optimal parameters for obtaining the minimal temperature through laser cooling of lithium atoms, it seems reasonable to employ a conventional single-particle approximation of the MOT model.

Note that the momentum distribution of atoms in a cooling laser field is essentially inhomogeneous [17] and, in the strict sense, cannot be described in terms of temperature. Thus, for a quantitative estimate of a laser cooling we will present an average kinetic energy of cold atoms. In Fig. 3, results are presented on the average kinetic energy for 6Li atoms reachable under laser cooling with the employment of the light field, which is in resonance with the optical transition ${}^{2}S_{1/2} \rightarrow {}^{2}P_{3/2}$ in a field with $\sigma_{+} - \sigma_{-}$ configuration. The results are obtained in the frameworks of the simplified and more complete models (Fig. 2) for various detunings of the light field $\delta = \omega - \omega_0$ [here, ω is the frequency of the light field, and ω_0 is that of the optical transition (Fig. 2)] as functions of the parameter, which determines the light shift of levels

$$
U = \frac{|\delta|}{3} \frac{\Omega^2}{\delta^2 + \gamma^2/2},\tag{4}
$$

Figure 3. (Colour online) Kinetic energy of cold ⁶Li atoms as a function of parameter *U* at various detunings in the field of σ_{+} – σ_{-} configuration in (a) simplified and (b) more complete models (Fig. 2); DL is the Doppler limit denoted by the horizontal dashed line.

where

$$
\Omega = \gamma \sqrt{\frac{I}{2I_s}} \tag{5}
$$

is the Rabi frequency of optical transition, which can be expressed in terms of the saturation intensity of the optical transition [35] and laser field intensity; *I* is the field intensity for each of the oncoming waves; and $I_s \approx 2.56$ mW cm⁻² is the saturation intensity for the used optical transition of lithium atoms [35].

It follows from Fig. 3 that the kinetic energy of atoms does not reach the Doppler limit of laser cooling $k_B T_D \sim$ $\hbar\gamma/2$ (\sim 140 μ K for lithium atoms), which corresponds to the average kinetic energy of cold atoms $E_{kin} \sim \hbar \omega_r$ (the limit is shown as a dashed horizontal line). This fact is confirmed qualitatively by experimental data: in paper [36], with a detuning of $\delta = -3\gamma$ at the first cooling stage, the laser cooling temperature of \sim 500 μ K was reached (which corresponds to the average kinetic energy $E_{kin} \sim 1.8\hbar\gamma \sim 70\hbar\omega_r$). A somewhat lower temperature of \sim 410 μ K ($E_{kin} \sim 1.5\hbar\gamma \sim 60\hbar\omega_r$) has been obtained [37] with a detuning $\delta = -0.5\gamma$. In [38], a nonstandard MOT model was suggested with a temperature of ~1000 μ K (E_{kin} ~ 3.6 $\hbar \gamma$ ~ 140 $\hbar \omega_r$) and detuning δ = $-3.4y$.

It worth noting substantial discrepancies between the results obtained with the simplified and more complete models (Fig. 3). In weak fields, the latter model, which takes into account a complicated structure of lithium atom levels in the excited state, predicts a deeper laser cooling as compared to the simplified model. However, in strong fields, an optimal intensity of the light field is not observed; the average kinetic energy monotonously increases with the intensity.

The employment of a lin \perp lin configuration for the light field formed by counterpropagating light waves with orthogonal linear polarisations allows one to perform deeper laser cooling as compared to the σ_{+} – σ_{-} configuration (Fig. 4). In this case, only quantitative differences are observed between these models in a large detuning regime. Importantly, despite of the deeper laser cooling this configuration of the light field cannot be used for realising a MOT, because a retaining magneto-optical potential is not produced.

For obtaining deep laser cooling with a retaining magneto-optical potential, let us consider the more general configuration of a light field realised by counterpropagating waves with elliptic polarisations (Fig. 5). In the coordinate system with the *z* axis directed along the wavevectors of the counterpropagating waves (1), polarisation vectors e_1 and e_2 can be presented as the expansion in terms of the circular components:

$$
e_1 = -\cos(\varepsilon_0 - \pi/4)e_+ + \cos(\varepsilon_0 + \pi/4)e_-,
$$

\n
$$
e_2 = -\cos(\varepsilon_0 + \pi/4)\exp(-i\theta)e_+ + \cos(\varepsilon_0 - \pi/4)\exp(i\theta)e_-.
$$
 (6)

Here, θ is the relative angle of the mutual orientation between principal semiaxes of the polarisation ellipses of the counterpropagating waves; and the parameter ε_0 characterising the ellipticity of the waves is equal to arctangent of the ratio of principal semiaxes of polarisation ellipse, $-\pi/4 < \varepsilon_0 < \pi/4$. This field configuration is denoted as $\varepsilon - \theta - \bar{\varepsilon}$ [39, 40]. The known configurations of the light field lin \perp lin ($\varepsilon_0 = 0, \theta = \pi/2$) and σ_+ – σ_- (ε_0 = π /4) are particular cases of the considered $\varepsilon - \theta - \bar{\varepsilon}$ configuration. The ellipticity parameter of the counterpropagating waves $\varepsilon_0 = 0$ corresponds to a linear

Figure 5. Configuration $\varepsilon - \theta - \bar{\varepsilon}$ of a light field formed by counterpropagating waves with opposite elliptic polarisations. The configuration is determined by the ellipticities of counterpropagating wave ε_0 and $-\varepsilon_0$.

Figure 4. (Colour online) Kinetic energy of cold ⁶Li atoms as a function of parameter *U* at various detunings δ in the field of lin \bot lin configuration in (a) simplified and (b) more complete models (Fig. 2); DL is the Doppler limit denoted by the horizontal dashed line.

Figure 6. (Colour online) Kinetic energy of cold ⁶Li atoms as a function of parameter *U* in the field of $\varepsilon - \theta - \bar{\varepsilon}$ configuration at various ellipticity parameters of counterpropagating waves ε_0 ; the angle is $\theta = \pi/2$ for (a) simplified and (b) more complete models. DL is the Doppler limit denoted by the horizontal dashed line.

polarisation, and $\varepsilon_0 = \pm \pi/4$ corresponds to a left- or righthand circular polarisation.

Analysis of laser cooling limits in the field of $\varepsilon - \theta - \bar{\varepsilon}$ configuration shows that in both models, in a transfer from a linear to circular polarisation ($|\varepsilon_0|$ increases), the minimal reachable average kinetic energy sharply grows (Fig. 6). Therefore, for obtaining deep sub-Doppler laser cooling of ⁶ Li atoms in the MOT operation conditions we suggest the employment of $\varepsilon - \theta - \bar{\varepsilon}$ configuration for the light field with a small ellipticity parameter ε_0 . In this case, unlike the lin \perp lin configuration, in the field with $\varepsilon - \theta - \bar{\varepsilon}$ configuration a magneto-optical force arises that retains atoms [39]. This force comprises polarisation contributions, which do not arise in the fields formed by oncoming waves with linear or circular polarisations.

Dependences of the average kinetic energy of cold ⁶Li atoms on the ellipticity parameter of the counterpropagating waves ε_0 at $\theta = \pi/4$ are given in Fig. 7. The limiting points $\varepsilon_0 =$ + π /4 and $\varepsilon_0 = -\pi$ /4 correspond to $\sigma_+ - \sigma_-$ and $\sigma_- - \sigma_+$ configurations of light fields, respectively, and point $\varepsilon_0 = 0$ refers to the $\text{lin}\angle\text{lin}$ configuration. The symmetry observed with respect to the ellipticity parameter is determined by anomalous polarisation contributions into a friction force [40].

Figure 7. (Colour online) Dependence of the kinetic energy of cold ⁶Li atoms in the field of $\varepsilon-\theta-\bar{\varepsilon}$ configuration ($\theta=\pi/4$) on the ellipticity parameter of the light wave ε_0 . DL is the Doppler limit denoted by the horizontal dashed line.

5. Magneto-optical potential in a MOT with field configuration $\varepsilon - \theta - \bar{\varepsilon}$

Calculation results for nonlinear dependence of the magnetooptical force in a MOT are presented in Fig. 8a at various ellipticity parameters of the light waves forming the trap. For comparison, $\sigma_+ - \sigma_-$ ($\varepsilon_0 = \pi/4$) and lin \angle lin- ($\varepsilon_0 = 0$) configurations are included. The sign for the angle $\theta = -\pi/4$ is chosen negative in order to make the magneto-optical force retaining, when atoms deviate from the equilibrium point to a position *z* > 0 with a nonzero magnetic field directed along the *z* axis. In this case, the magneto-optical force generates a trapping magneto-optical potential in a MOT.

Dependences of the depth of the magneto-optical potential on the value of the magnetic field are given in Fig. 8b for σ_+ – σ_- and $\varepsilon - \theta - \bar{\varepsilon}$ configurations of the light field. The depth of the magneto-optical potential in the field with the $\varepsilon - \theta - \bar{\varepsilon}$ configuration at the light field ellipticity parameters considered is 6 –10 times less than that of the standard MOT, formed by the σ_{+} – σ_{-} configuration of light fields; nevertheless, the corresponding temperature remains substantially higher than the temperature of trapped cold atoms. For example, in a MOT formed by the fields with $\varepsilon - \theta - \bar{\varepsilon}$ configuration of size $z = 0.5$ cm with the magnetic field gradient $\partial_z H = 6$ Gs cm⁻¹, the depth of the magneto-optical potential (Fig. 8b) $U_{\text{MOT}} = 1000 \hbar \gamma \approx 0.3 \text{ K}$, which is greater by more than three order in magnitude than the Doppler limit for lithium atoms $(T = 140 \mu K)$ and is quite sufficient for retaining a cloud of cold atoms. In this case, the reached temperature for laser cooling of lithium atoms in the suggested configuration of a light field is substantially less (lower than the Doppler limit) (see Fig. 7), which is an incontestable advantage of the suggested MOT configuration for obtaining deep laser cooling of lithium atoms.

6. Conclusions

A magneto-optical trap formed by counterpropagating waves with elliptic polarisations is proposed for deep cooling of lithium atoms. In the frameworks of the simplified model (a twolevel system with levels degenerated with respect to the angular momentum projection) and more complete model (taking into account a hyperfine structure of the excited atomic state),

Figure 8. (Colour online) (a) Dependence of the magneto-optical force F_{MOT} (in units of *ky*) acting on ⁶Li atoms on the value of the magnetic field *H* and (b) dependence of the depth of magneto-optical potential Γ_{MOT} (in units of *y*) on MOT dimension (gradient of magnetic field is 6 Gs cm⁻¹) in the field of $\varepsilon-\theta-\bar{\varepsilon}$ configuration at various ellipticity parameters of counterpropagating waves ε_0 . The detunings and intensities of light waves correspond to the optimal parameters for light fields shown in Fig. 7.

we comparatively analysed the limits of laser cooling of lithium atoms in a field with the most general spatial configuration $\varepsilon - \theta - \bar{\varepsilon}$.

It is shown that a MOT formed by elliptically polarised waves allows one to obtain deeper laser cooling for 6Li atoms as compared to the standard MOT based on counterpropagating circularly polarised waves. Thus, in the field with $\varepsilon - \theta - \bar{\varepsilon}$ configuration formed by the elliptically polarised waves and oriented at the angle $\theta = -\pi/4$ with the ellipticity parameters close a linear polarisation, it is possible to reach the average kinetic energy of cold atoms $E_{kin} \sim 20 \hbar \omega_r$, which for the equilibrium distribution corresponds to the temperature of laser cooling $T \approx 140 \mu K$. As compared to a MOT formed by waves with circular polarisations σ_{+} – σ_{-} , the MOT with a field of $\varepsilon - \theta - \bar{\varepsilon}$ configuration exhibits a lower, however, sufficient for trapping potential depth and allows one to reach lower temperatures in laser cooling of ⁶Li atoms. The employment of a MOT formed by elliptically polarised waves seems promising for the second stage of deep laser cooling of lithium atoms.

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