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INTERACTION OF LASER RADIATION WITH MATTER. LASER PLASMA https://doi.org/10.1070/QEL17990

Synchrotron radiation from a cluster plasma in a circularly polarised laser field

A.A. Andreev, K.Yu. Platonov

Abstract. **An analytical model is developed for the generation of synchrotron radiation from a laser cluster plasma in the focal waist of an ultra-intense short circularly polarised laser pulse. The rotation of relativistic electrons around the ionised core of the cluster with a radius of rotation smaller than the laser wavelength leads to intense synchrotron radiation in the direction transverse to the laser wave vector. The parameters of the cluster plasma and laser pulse are determined at which, due to the small radius of curvature of the electron trajectory of the cluster shell, the intensity of synchrotron radiation exceeds the intensity of betatron radiation of the electron flux in the longitudinal (along the wave vector) direction.**

Keywords: *cluster plasma, circularly polarised radiation, relativistic intensity.*

1. Introduction

In our previous papers [1–4], we developed a theory of generation of ultra-intense magnetic fields and giant magnetic moments, based on electron inertia in cluster gas targets irradiated by an ultra-intense short circularly polarised laser pulse of relativistic intensity. A toroidal long-lived relativistic electric current in the shells of clusters generates synchrotron radiation, the parameters of which were not estimated in the above studies, since the synchrotron energy losses of electrons in the considered range of laser intensities are small compared to other energy losses. At the same time, a cluster gas target generates secondary hard radiation through several (in addition to synchrotron) physical mechanisms. Thus, in addition to hot electrons localised on clusters, the gas target produces an electron flux propagating in the plasma channel formed in the focal waist of the laser pulse. A flux of such fast electrons from a transparent background plasma generates betatron radiation due to the oscillations of electrons in the electrostatic field of the plasma channel. This radiation has been considered in a large number of works (see, for example, papers [5, 6] and references therein); it has a high intensity and directivity (along the wave vector of the laser wave). The addition of clusters with a radius of several nanometres to the gas target (the field strength of the ion core for such clusters is lower than the laser field strength) serves as an additional source of electrons and an additional channel for their scattering, increasing the betatron radiation intensity by two to three orders of magnitude compared to a gas target without clusters [7]. The clusters of larger radii (tens and hundreds of nanometres) considered in this work, the electrostatic potential of which is sufficient to confine the electron shell rotating in a circularly polarised laser field, are also an additional channel for electron beam scattering, and increase the betatron radiation intensity.

Apart from synchrotron and betatron radiation mechanisms, bremsstrahlung is generated in the laser plasma, the parameters of which are determined by the density and temperature of hot electrons. Note that bremsstrahlung is a process of a higher order of smallness in the electron charge and dominates over synchrotron and betatron radiation only in those spectral ranges where these radiations are suppressed. This is where the question arises: Which of the secondary radiation channels dominates (has the highest radiation energy) and what are the conditions for the prevalence of the previously unexplored synchrotron channel over the other channels? This work is devoted to calculating the intensity of synchrotron radiation from a cluster laser plasma and finding the conditions (parameters of clusters and laser pulses) under which the synchrotron radiation channel dominates.

Note that the synchrotron and other radiation channels have different (across and along the wave vector of the laser wave) angular directivity; therefore, the synchrotron channel can be distinguished against the background of the others even at a low radiation intensity of this channel. Due to the small radius of curvature of the trajectory of an electron in the cluster shell as compared to the radius of curvature of the trajectory of an electron performing betatron oscillations, the intensity of synchrotron radiation exceeds the intensity of betatron radiation for a single particle. However, the ratio between the number of electrons localised near the clusters and the number of electrons propagating in the direction of the laser pulse, as well as their characteristic energies, depend on the parameters of the gas cluster target and the laser pulse; therefore, depending on the conditions, any of the three channels listed above may dominate. In addition, any of the channels of secondary hard radiation under our conditions is incoherent in the number of emitting electrons [8], which contradicts the results of [9]; therefore, synchrotron radiation of the cluster shell is described by the standard theory of synchrotron radiation [10].

A.A. Andreev St. Petersburg State University, Universitetskaya nab. 7–9, 199034 St. Petersburg, Russia; Ioffe Institute, Polytekhnicheskaya ul. 26, 194021 St. Petersburg, Russia;
K.Yu. Platonov Peter the Great St. Petersburg Polytechnic University, Polytechnicheskaya ul. 29, 195251 St. Petersburg, Russia;
e-mail: konstantin_platonov@yahoo.com

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2. Dynamics of cluster electrons and intensity of secondary radiation

When a spherical cluster is irradiated by a circularly polarised field, there arises an orbital relativistic electron current, which exists during the entire lifetime (expansion) of the cluster (Fig. 1). Secondary radiation is generated by rotating electrons both during the action of a laser pulse of duration $\tau_{\rm L}$ (incoherent scattering of the laser field $E_{\rm L}$) and after the termination of the pulse due to electron inertia (motion of electrons in quasi-stationary electric and magnetic fields of the cluster). The radiation energy is determined by the square of the external field strength (magnetic H_Q or electric E_Q) and the time of its action on the electron [10]. Consequently, if the eigenfields H_Q and E_Q of the cluster and its lifetime $\tau_{\rm cl}$ are such that $(H_Q^2 + E_Q^2)\tau_{\rm cl} > E_{\rm L}^2\tau_{\rm L}$, then the main contribution to the secondary radiation is made by the inertia of electrons. In this paper, we consider this case. According to PIC calculations [11], the ratio H_Q/E_Q can exceed unity and increases with increasing laser intensity; therefore, relativistic electrons emit mainly due to the magnetic field of the cluster. We will call such radiation synchrotron radiation. In the absence of a quasi-stationary magnetic field (for example, in the case of a linearly polarised laser field incident on the cluster), the emission of cluster electrons occurs in the quasi-stationary electric field E_Q of the ionic core of the cluster. This radiation will be called betatron radiation. When $H_Q \approx E_Q$, we will use the term secondary radiation.

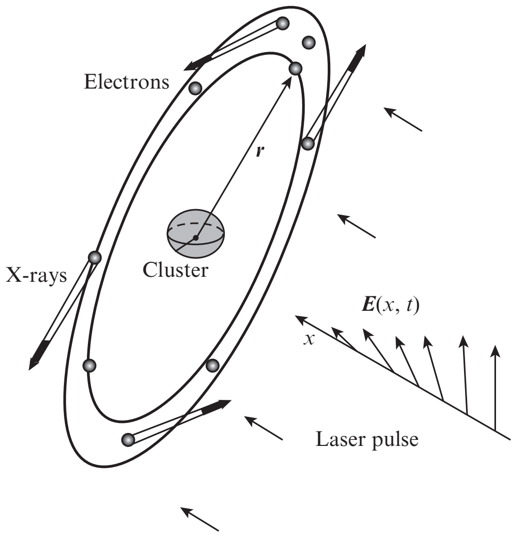

Figure 1. Scheme of generation of synchrotron radiation from the rotating electron shell of the cluster.

Let us consider the dynamics of an electron in quasi-stationary electric and magnetic fields of a cluster with an initial radius R_0 and in the field of circularly polarised laser radiation $\boldsymbol{E}(x,t) = E_{\rm L}\cos(kx - \omega t)\boldsymbol{e}_{\rm y} + E_{\rm L}\sin(kx - \omega t)\boldsymbol{e}_z$ with an intensity $I = cE_{\rm L}^2/4\pi$ and a pulse duration $\tau_{\rm L}$ (i.e., on the time interval $0 < t < \tau_{\rm L}$). The vector potential of such a wave has the form

$$\boldsymbol{A}(x,t) = (E_0/k)[\sin(kx - \omega t)\boldsymbol{e}_y - \cos(kx - \omega t)\boldsymbol{e}_z]. \quad (1)$$

The Lagrange function of a shell electron in the electromagnetic fields of a cluster and a laser pulse in a cylindrical coordinate system with the x axis directed along the axis of a circularly polarised laser beam has the form:

$$L = -m_{\rm e}c^2/\gamma + \frac{e}{c}\left[A_\alpha(r,x) - \frac{E_{\rm L}}{k}\cos(kx - \omega t - \alpha)\right]r\dot{\alpha}$$
$$+ \frac{eE_{\rm L}}{\omega}\dot{r}\sin(kx - \omega t - \alpha) - e\varphi(r,x), \quad (2)$$

where $m_{\rm e}$ is the electron mass; $\gamma = 1/\sqrt{1-\beta^2}$; $(\beta c)^2 = \dot{x}^2 + \dot{r}^2 + (r\dot{\alpha})^2$; and $e < 0$ is the electron charge.

The scalar potential

$$\varphi(r,x) = \frac{Q}{(r^2 + x^2)^{1/2}} - \frac{Q(r^2 + x^2 + p^2)}{2p^2},$$

where $R_0 \leqslant (r^2 + x^2)^{1/2} \leqslant p$, corresponds to the electric field of the ionic core with radius R_0 and the electron shell with radius p and vanishes (screening) at $r^2 + x^2 = p^2$. The angular component of the vector potential of a quasi-stationary magnetic field for a uniform spherical layer rotating with an angular velocity $\dot{\alpha}$ and having a charge Q has the form

$$A_\alpha(r,x) = -\frac{Q\dot{\alpha}r}{2cp}\left[1 - \frac{3(r^2 + x^2)}{5p^2}\right],$$
$$R_0 \leqslant (r^2 + x^2)^{1/2} \leqslant p. \quad (3)$$

The electron cluster radius p (the radius corresponding to the critical electron density $n_{\rm cr}$) differs from the initial cluster radius R_0 by several times. Indeed, let the laser pulse heat all the electrons of the cluster (which is an upper estimate for p, since the skin-layer length $l_{\rm s}$ is less than R_0). Then, at an initial electron density of a cluster equal to $\sim 100 n_{\rm cr}$, the value of p is assessed as $p \approx R(100)^{1/3} \approx 4.6R$. Thus, $R_0 < p < 4.6R_0$, and for subsequent estimates one can choose $p \approx (3-4)R_0$ according to the best fit between the model and numerical calculation. The laser energy absorbed by the cluster is $E_{\rm abs} \approx \eta I\pi p^2\tau_{\rm L}$. The absorption coefficient η of a cluster laser target has been investigated in many works (see, for example, [12–14]). In particular, Gozhev et al. [12] obtained an estimate formula for η as a function of the laser intensity; for $I \to \infty$ this formula has a limit $\eta \to \eta_0 = 1/3$. However, estimates based on the formulae from [13, 14] give a higher absorption coefficient; therefore, in what follows we will consider $\eta_0 \in [0.3, 0.5]$, and the dependence of the cluster absorption coefficient on the laser intensity will be taken from [12]. The energy absorbed by the cluster is related to the absorbed total mechanical moment by the ratio $J_{\rm abs} = E_{\rm abs}/\omega$. From this we obtain $J_{\rm abs} = \eta I\pi p^2\tau_{\rm L}/\omega$. When an electron rotates with an angular velocity ω along a circle of radius p, its characteristic mechanical moment is $M = \gamma m_{\rm e}p^2\omega$. The ratio $J_{\rm abs}/M$ is an estimate of the number of electrons in the cluster shell, $N_{\rm e} \approx \eta I\pi\tau_{\rm L}/(\gamma m_{\rm e}\omega^2)$, and the charge of the ionic core of the cluster, $Q = eN_{\rm e}$, introduced in (2), (3). μ The Lorentz factor of an electron is also estimated from the characteristic energy of the transverse motion of an electron in the wave field: $\gamma \approx \sqrt{1 + a^2}$, where $a = \sqrt{I\lambda^2/1.37\times10^{18}\,{\rm W\,\mu m^2\,cm^{-2}}}$ is the normalised vector potential. Thus, the parameters p and Q introduced into the Lagrange function are determined by the laser pulse parameters, the absorption coefficient, and the initial radius of the cluster.

The equation of motion of an electron along the angle α (the equation for the angular momentum of an electron) takes the form

$$\frac{\mathrm{d}}{\mathrm{d}t}\left\{\gamma m_{\mathrm{e}} r^2\dot{\alpha} - \frac{eQ\dot{\alpha}r^2}{2c^2p}\left[1 - \frac{3(r^2+x^2)}{5p^2}\right]\right\}$$

$$= eE_{\mathrm{L}}r(t)\left[1 - \dot{x}(t)/c\right]\sin\left[kx(t) - \omega(t) - \alpha(t)\right]. \quad (4)$$

The radial equation of motion of an electron

$$\frac{\mathrm{d}}{\mathrm{d}t}(\gamma m_{\mathrm{e}}\dot{r}) = \gamma m_{\mathrm{e}} r\dot{\alpha}^2 + \frac{eQr}{(r^2+x^3)^{3/2}}\left[1 - \frac{\dot{\alpha}^2(r^2+x^3)^{3/2}}{c^2p}\right]$$

$$+ eE_{\mathrm{L}}\left(1 - \frac{\dot{x}}{c}\right)\cos\left[kx(t) - \omega(t) - \alpha(t)\right] \quad (5)$$

shows that the radial motion occurs under the action of the centrifugal force $\gamma m_{\mathrm{e}} r\dot{\alpha}^2$, the force of the Coulomb interaction with the ionic core, reduced by the value of the Lorentz force of the quasi-stationary magnetic field [these two forces correspond to the second term on the right-hand side of (5)], and the force acting from the side of the laser wave [(the last term in (5)]. The equation of motion of an electron in the longitudinal (along the x axis) direction, as well as equations (4) and (5), follows from expression (2):

$$\frac{\mathrm{d}}{\mathrm{d}t}(\gamma m_{\mathrm{e}}\dot{x}) = -\frac{\partial}{\partial x}\Big[-e\frac{r\dot{\alpha}}{c}A_{\alpha}(r,x) + e\varphi(r,x)$$

$$+\frac{eE_{\mathrm{L}}}{\omega}\cos(kx - \omega t - \alpha)r\dot{\alpha} - \frac{eE_{\mathrm{L}}}{\omega}\dot{r}\sin(kx - \omega t - \alpha)\Big]. \quad (6)$$

Equations (4)–(6) can be solved numerically and the laws of motion $\alpha(t)$, $r(t)$, $x(t)$ can be found both during the action of the laser pulse and after its termination ($E_{\mathrm{L}} = 0$, $t > \tau_{\mathrm{L}}$). In solving the equations, one needs to introduce the initial velocity (chosen to be zero) and the initial coordinates $\boldsymbol{r}_0 = (r_0, \alpha_0, x_0)$ of the electron, belonging to the intervals $0 \leqslant \alpha_0 < 2\pi$ and $R_0^2 \leqslant x_0^2 + r_0^2 \leqslant p^2$. In subsequent calculations of the radiation energy over the initial coordinates, an averaging is carried out corresponding to the uniform filling of the cluster shell with electrons. Accordingly, the initial distribution function of electrons over coordinates has the form

$$f(x_0, \alpha_0, r_0) = 3N_{\mathrm{e}}\Theta(x_0^2 + r_0^2 - R_0^2)$$

$$\times\Theta(p^2 - x_0^2 - r_0^2)/[8\pi^2(p^3 - R_0^3)],$$

where $\Theta(x)$ is the Heaviside step function.

In addition to the numerical solution, it is possible to construct an approximate solution of the equations of dynamics by the averaging method. If equation (5) is averaged over several laser field cycles [which corresponds to the search for a solution in the form $r(t) = \langle r\rangle + \delta r(t)$, $\dot{\alpha}(t) = \langle\dot{\alpha}\rangle + \delta\dot{\alpha}(t)$, $x(t) = \langle x\rangle + \delta x(t)$], then the average values of the total derivative on the left-hand side of (5) and the force acting from the laser field will vanish and the time-averaged radial motion will be determined by the compensation of the centrifugal and 'Coulomb' forces acting on an electron:

$$\gamma m_{\mathrm{e}}\langle r\rangle\langle\dot{\alpha}\rangle^2 + \frac{eQ\langle r\rangle}{\left(\langle r\rangle^2 + \langle x\rangle^2\right)^{3/2}}$$

$$\times\left[1 - \frac{\langle\dot{\alpha}\rangle^2\left(\langle r\rangle^2 + \langle x\rangle^3\right)^{3/2}}{c^2p}\right] = 0. \quad (7)$$

Since the rotation of an electron is caused by the action of a circularly polarised wave and the magnetic field of the cluster, the average value is $\langle\dot{\alpha}\rangle \approx -\omega \approx -\omega_H = -eH_Q/(\gamma m_{\mathrm{e}}c)$ [the minus sign appeared because the direction of rotation of the wave electric field vector $\boldsymbol{E} = E_0\cos(kx - \omega t)\boldsymbol{e}_y + E_0\sin(kx - \omega t)\boldsymbol{e}_z$ corresponds to rotation from the z axis to the y axis, which is opposite to the direction of increasing the angle α because it is measured from the y axis to the z axis]. The quantity $\langle r\rangle$, up to a numerical factor of the order of unity, coincides with the previously introduced electron radius of the cluster: $\langle r\rangle \approx p$. Thus, $r(t) \approx p$, $\alpha(t) \approx -\omega_H t$. Finding the solution to Eqn (6) in the form $x(t) = \langle x\rangle + \delta x(t)$, we can write the equation for $\langle x\rangle$ corresponding to the equality of the force of the ponderomotive pressure of the laser wave to the force of the quasi-stationary electric field incident on the electron in the longitudinal direction:

$$\frac{eQ\langle x\rangle}{\left(p^2 + \langle x\rangle^2\right)^{3/2}}\left(1 + \frac{3p^2\omega^2}{5c^2}\right) + \frac{eE_{\mathrm{L}}p\omega}{c}\sin(k\langle x\rangle) = 0. \quad (8)$$

In the weak-field approximation ($eE_{\mathrm{L}}/(m_{\mathrm{e}}\omega c) \ll 1$), we have $\langle x\rangle \approx 0$, which corresponds to the minimum of the electrostatic potential energy at the centre of the ionic core. Note that Eqn (8) corresponds to the condition for a minimum of the effective longitudinal potential energy of an electron

$$U_{\mathrm{eff}}(x) = -epkA_{\alpha}(p,x) + e\varphi(p,x) + eE_{\mathrm{L}}p\cos(kx)$$

$$\approx epk^2Q/5 + eE_{\mathrm{L}}p - ek^2x^2Q$$

$$\times[0.3 + k^{-2}p^{-2} + eE_{\mathrm{L}}p^2/(2Q)]/p,$$

coinciding with the energy of the harmonic oscillator (for $e < 0$). The oscillating addition $\delta x(t)$ satisfies the equation of oscillations of a relativistic harmonic oscillator:

$$\frac{\mathrm{d}}{\mathrm{d}t}(\gamma m_{\mathrm{e}}\delta\dot{x}) = -\frac{\partial U_{\mathrm{eff}}\left(\langle x\rangle + \delta x\right)}{\partial\langle x\rangle}$$

$$\approx -\delta x\frac{\partial^2 U_{\mathrm{eff}}\left(\langle x\rangle\right)}{\partial\langle x\rangle^2} = -\delta x\frac{|e|Qk^2}{p}\left(\frac{3}{10} + \frac{1}{kp} + \frac{E_{\mathrm{L}}p^2}{2Q}\right). \quad (9)$$

Thus, the motion of an electron in a cluster is the rotation of an electron in the transverse plane yz and its simultaneous vibration along the x axis relative to the value of $\langle x\rangle$ with a frequency

$$\Omega_x = \omega\sqrt{\frac{|e|Q}{p\gamma m_{\mathrm{e}}c^2}\left(\frac{3}{10} + \frac{1}{kp} + \frac{E_{\mathrm{L}}p^2}{2Q}\right)}.$$

In the case of a finite motion of a cluster electron, its potential energy eQ/p is of the order of the kinetic energy $\gamma m_e c^2$ and $\Omega_x \approx \omega$. This character of the motion is confirmed by the numerical PIC simulation performed in [2], where the trajectories of the electron are presented. Obviously, the approximate solution corresponds to the consideration of an 'averaged' electron, i.e., to the same laws of motion for all electrons in the cluster shell. The radiation energy calculated for one such particle must be multiplied by the number of electrons N_e to estimate the radiation intensity of the entire cluster shell. The laws of electron motion $\alpha(t, \mathbf{r}_0)$, $r(t, \mathbf{r}_0)$, $x(t, \mathbf{r}_0)$ found from Eqns (4)–(6) make it possible to calculate the secondary radiation energy of cluster electrons in its quasi-stationary electric and magnetic fields [10]:

$$\varepsilon_\beta = \frac{2e^4}{3m_e^2 c^3} \int f(\mathbf{r}_0)\, \mathrm{d}^3\mathbf{r}_0$$

$$\times \int_{-\infty}^{\infty} \mathrm{d}t \left\{ \left[\mathbf{E}_Q(x(t,\mathbf{r}_0); r(t,\mathbf{r}_0)) + \frac{1}{c}\dot{\mathbf{r}}(t,\mathbf{r}_0) \times \mathbf{H}_Q(x(t,\mathbf{r}_0); r(t,\mathbf{r}_0)) \right]^2 \right.$$

$$\left. - \frac{1}{c^2}\left(\mathbf{E}_Q(x(t,\mathbf{r}_0); r(t,\mathbf{r}_0))\, \dot{\mathbf{r}}(t,\mathbf{r}_0) \right)^2 \right\} \left[1 - \frac{\dot{\mathbf{r}}^2(t,\mathbf{r}_0)}{c^2} \right]^{-1}, \qquad (10)$$

where

$$\mathbf{E}_Q(x,r) = -\frac{\partial\varphi}{\partial x}\mathbf{e}_x - \frac{\partial\varphi}{\partial r}\mathbf{e}_r;$$

$$\mathbf{H}_Q(x,r) = \mathbf{e}_x \frac{Q\omega}{2cp}\left[1 - \frac{3(r^2 + x^2)}{5p^2} \right].$$

Note that the secondary radiation of electrons in the cluster shell is incoherent in the number of electrons [$f \propto N_e$ in (10)] due to the assumed uniform filling of the cluster shell with electrons. For the manifestation of coherence in the cluster shell, there must be electron bunches (density inhomogeneities) with a spatial size of the order of the wavelength of the secondary radiation. For the energies of secondary quanta from units to tens of keV, these are sizes from tenths to hundredths of a nanometre. The issue of the existence of such bunches requires additional study and will be considered elsewhere. We only note that the rotation of bunches in the shell of the cluster during a laser pulse will cause the appearance of coherent pulses of zeptosecond duration ($\sim\omega^{-1}\gamma^{-3}$). In this paper, we consider the time interval after the termination of a short pulse, when the fluctuations of the electron density of the cluster shell caused by the laser field should be smoothed out. Formula (10) is written for a constant value of the characteristic radius p of the electron orbit without taking into account the expansion of the ionic core of the cluster. If we take it into account, it is necessary to make the replacement

$$p \to p(t) \approx p + ct\sqrt{Zm_e(\gamma - 1)/(Am_p)},$$

where Z and A are the charge and atomic number of the cluster ion, and m_p is the proton mass. Accordingly, the characteristic lifetime of the cluster is

$$\tau_{cl} \approx p / \left[c\sqrt{Zm_e(\gamma - 1)/(Am_p)} \right].$$

For heavy clusters (Au, Xe) with a radius of hundreds of nanometres, the value of τ_{cl} turns out to be of the order of hundreds of femtoseconds. In formula (10), the expansion of the cluster is taken into account by introducing a finite interval of integration over time ($\sim\tau_{cl}$), which is equivalent to the replacement $p \to p(t)$. The distribution of electrons over the initial coordinates and the equations of electron motion (4)–(6) in the quasi-stationary fields of the cluster do not take into account the inverse effect of synchrotron radiation on the cluster dynamics; therefore, $\varepsilon_\beta \ll N_e m_e c^2(\gamma - 1)$.

The integrals in (10) can be calculated by numerical methods or approximately (by the mean value theorem). In the latter case, we obtain an obvious estimate of the secondary radiation energy of the cluster in order of magnitude:

$$\varepsilon_\beta \approx \frac{2e^4 N_e \gamma^2 \tau_{cl}}{3m_e^2 c^3}(H_Q^2 + E_Q^2),$$
$$H_Q \approx \frac{Q\omega}{2cp}, \quad E_Q \approx \frac{Q}{p^2}. \qquad (11)$$

The energy of secondary radiation quanta using formula (11) can be estimated for $I = 5.6 \times 10^{20}$ W cm^{-2}, $\tau_L = 6$ fs, Au^{+30} cluster with a radius of 100 nm, using the results of numerical simulation of laser cluster plasma from [11]. In this case, the cluster's magnetic field exceeds the electric one, $H_Q > E_Q$, and the radiation is synchrotron radiation. For the selected parameters, the number of shell electrons is $N_e \approx \eta I \pi \tau_L / (\gamma m_e \omega^2) \approx 8 \times 10^8$, the magnetic field is $H_Q \approx Q\omega/(2cp) \approx 15$ kT, and the electron gyrofrequency is $\omega_H = eH_Q/(\gamma m_e c) \approx 8 \times 10^{14}$ s^{-1}. The maximum of the synchrotron radiation spectrum falls on the quantum energy

$$\varepsilon = \frac{e\hbar H_Q \gamma^2}{m_e c} = \hbar\omega \frac{\eta a^3}{16} \frac{c\tau_L}{R_0} \approx 1.8 \text{ keV}. \qquad (12)$$

The spectrum itself has a form standard for synchrotron radiation [10]. At quantum energies exceeding ε (12), the spectrum exponentially decays to the level of the bremsstrahlung background (Bethe–Heitler radiation caused by scattering of electrons by cluster ions). The estimate of the total energy emitted by the cluster by formula (11) yields

$$\varepsilon_\beta = N_e \frac{2e^4 H_Q^2 \tau_{cl}}{3m_e^2 c^3}\gamma^2$$
$$= \frac{\sqrt{Am_p m_e}\, c^2}{24\sqrt{Z}} \frac{\eta^3 a^{4.5} c\tau_L^3 \omega^2}{R_0} \approx 47 \text{ erg}. \qquad (13)$$

The number of quanta is

$$N_\beta = N_e \frac{2e^3 H_Q \tau_{cl}}{3\hbar m_e c^2}$$
$$= \frac{\eta^2 a^{3/2} \sqrt{Am_p m_e}\, \omega c^2 \tau_L^2}{24\hbar\sqrt{Z}} \approx 1.6 \times 10^{10}. \qquad (14)$$

The laser pulse energy ε_{Lp} absorbed by the shell at the absorption coefficient $\eta_0 = 0.5$ will be $\eta I \tau_L \pi p^2 = I\tau_L \pi (4R_0)^2 = 5 \times 10^4$ erg. Accordingly, the conversion coefficient, defined as the ratio of the synchrotron radiation energy to the laser energy absorbed by the cluster, is estimated by the formula

$$\kappa = \frac{\varepsilon_\beta}{\varepsilon_{\mathrm{Lp}}} = \frac{\sqrt{Am_\mathrm{p}}}{96\sqrt{Zm_\mathrm{e}}}\frac{\eta^3 a^{2.5} e^2 \tau_\mathrm{L}^2}{m_\mathrm{e} R_0^3} \approx 10^{-3}. \qquad (15)$$

The angular energy (intensity) distribution of synchrotron radiation of the cluster is a ‘disk’ in the direction transverse to the wave vector of the laser wave with a characteristic angle of emission of a quantum $\theta \approx \gamma^{-1} \approx 5°$ relative to the disk plane. Note that the upper bound for the total energy of bremsstrahlung (Bethe–Heitler) radiation has the form $\varepsilon_{\mathrm{BH}} \approx N_\mathrm{e}(4/137)\times Z^2 c\tau_{\mathrm{cl}} n_\mathrm{i}[e^4/(m_\mathrm{e}c^2)](\gamma - 1)(\ln 2\gamma - 1/3)$, where $n_\mathrm{i} \approx 6\times 10^{22}$ cm^{-3} is the characteristic density of cluster ions. Even with such an overestimated estimate (when all hot electrons move in a medium with a density n_i, which is not true for a cluster), $\varepsilon_{\mathrm{BH}} \approx 10^{-2}\varepsilon_\beta$; therefore, the bremsstrahlung channel manifests itself only in the range of quantum energies $\varepsilon \gg \omega_H\gamma^3$. Estimates (12)–(15) did not take into account the electric field of the cluster, $E_Q \approx Q/p^2$, which is responsible for the longitudinal vibrations of the electron along the x axis. Let us show that this approximation is justified for the used parameters of the cluster and laser pulse. Estimates by the above formulae (for $p = 4R_0$) yield

$$\frac{H_Q}{E_\mathrm{L}} = \frac{e\eta E_\mathrm{L}\tau_\mathrm{L}}{4m_\mathrm{e}\omega\gamma p} = \eta\frac{a}{\sqrt{1+a^2}}\frac{c\tau_\mathrm{L}}{4p} \approx 0.56,$$

$$\frac{E_Q}{E_\mathrm{L}} = \frac{eN_\mathrm{e}}{16R_0^2 E_\mathrm{L}} \approx 0.16. \qquad (16)$$

These values are confirmed by the data of PIC simulation of the cluster fields [1, 2]. Thus, the contribution of the magnetic field to the energy of secondary radiation (11) is $(0.56/0.16)^2 \approx 12$ times greater than the contribution of the electric field of the cluster, and the main part of the secondary radiation is synchrotron radiation, which is generated in the transverse direction, while the energy of the betatron radiation is an order of magnitude less in the longitudinal direction.

Formulae (12)–(15) estimate the parameters of the secondary radiation generated after the termination of the laser pulse. During the action of the laser pulse, the laser fields have the highest strengths, but due to the short (6 fs) laser pulse duration compared to the cluster lifetime $\tau_{\mathrm{cl}} \approx 120$ fs, the ratio $H_Q^2\tau_{\mathrm{cl}}/(E_\mathrm{L}^2\tau_\mathrm{L}) \approx 5$. Accordingly, the main contribution to the energy of the secondary radiation is made by synchrotron radiation of electrons after the termination of the laser pulse. Note that in the case of linear polarisation of the laser pulse, only betatron radiation remains in the electric field E_Q of the cluster. Estimation formulae (12)–(15) contain simple power-law dependences of the characteristics of the secondary radiation on the intensity of the laser pulse and its duration τ_L, which makes it possible to construct the corresponding linear dependences on a doubly logarithmic scale. The conversion coefficient (15) and the characteristic quantum energy (12) for an Au^{+30} cluster with a radius of 50 nm irradiated with an 8-fs pulse (for $\eta_0 = 0.3$) as functions of the laser intensity in the range 10^{19}–10^{21} W cm^{-2} are shown in Fig. 2. At high intensities and durations of a laser pulse, it is possible to achieve a conversion coefficient $\kappa \to \eta_0$, which means that it is necessary to take into account radiation losses when deriving the law of motion of an electron. Power-law expressions (12)–(15) have a sharp dependence (large exponents) on a and τ_L, but the maximum values of these laser parameters are limited by the conditions $\tau_\mathrm{L} < \tau_{\mathrm{cl}}$ (the cluster should not have time to fly apart during the laser pulse action) and a $a \leqslant a_{\mathrm{th}} = 2Ze^2 n_\mathrm{i} R_0\lambda/(3m_\mathrm{e}c^2)$. The threshold value a_{th} of the dimensionless laser field amplitude corresponds to the Coulomb explosion of the cluster (the detachment of all electrons by the laser field and the disappearance of the synchrotron channel of the secondary radiation). For the cluster in Fig. 2, the threshold a_{th} corresponds to an intensity of 4×10^{21} W cm^{-2}.

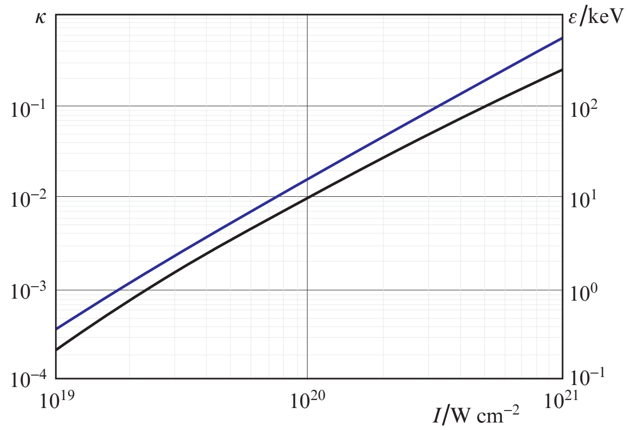

Figure 2. (Colour online) Coefficient of conversion of the laser energy absorbed by the cluster into the energy of synchrotron radiation (black curve and left ordinate) and the characteristic energy of a synchrotron radiation quantum (blue curve and right ordinate) as functions of laser intensity for an Au^{+30} cluster with a radius of 50 nm irradiated with a 8 fs pulse. The limiting absorption coefficient is $\eta_0 = 0.3$.

Note that in a real experimental situation, in addition to the presence of many (rather than a single) clusters, the space between the clusters is filled with transparent plasma (due to ionisation of the gas jet transporting the clusters to the focal region of the laser pulse). At a sufficiently high ($n_{\mathrm{ew}} = 10^{19}$–$10^{20}$ cm^{-3}) electron density of such a background plasma, it is possible to generate a wake plasma wave by a laser pulse and to accelerate electrons of a transparent plasma by this wave. In this case, betatron radiation of background plasma electrons directed along the wave vector of the laser wave [5, 6] arises in addition to the emission of cluster electrons in the transverse (due to the magnetic field H_Q) and longitudinal (due to the electric field E_Q) directions. The acceleration of background plasma electrons in a plasma wave weakly depends on the polarisation of the laser pulse and takes place for a circularly polarised pulse [15, 16]. To assess such an experimental situation, it is necessary to take into account the cluster density n_{cl} and the background plasma density between new clusters. The characteristic energy of an electron accelerated by a plasma wave is $\gamma_\mathrm{w} \approx 2a\omega^2/(3\omega_{\mathrm{pw}}^2)$ [17], where ω_{pw} is the plasma frequency corresponding to the electron density n_{ew}. The characteristic value of the radius of curvature of the trajectory of an electron during betatron oscillations is $\sim c/\omega_{\mathrm{pw}}$. The energy of betatron radiation of electrons of a transparent plasma is estimated by formula (11), which will include ω_{pw}^2 instead of the square of the cyclotron frequency $\omega_H^2 = e^2H_Q^2/(m_\mathrm{e}^2c^2\gamma^2)$. There is no analytical formula for the total number N_{ew} of electrons trapped in the plasma channel and accelerated by the plasma wave; however, in [18], a PIC simulation of the total charge $eN_{\rm ew}$ of accelerated electrons was performed in a wide range of laser intensities and background plasma densities. Based on the data of this work, we define the following power-law dependence: $N_{\rm ew} \approx 5\times 10^8 a^{1.3}(n_{\rm ew}/n_{\rm cr})^{-0.03}$, where $(n_{\rm ew}/n_{\rm cr}) \in [0.02; 1]$, and $a \in [3; 70]$. The characteristic time of betatron radiation is the time of flight through the focal volume (Rayleigh length) $\tau_{\rm w} \approx \omega r_{\rm L}^2/c^2$, where $r_{\rm L}$ is the laser beam radius. The characteristic quantum energy of betatron radiation is $\hbar\omega_{\rm pw}\gamma_{\rm w}^3$. As a result of the above, the energy of betatron radiation of electrons of transparent plasma can be estimated by the formula

$$\varepsilon_{\rm w} \approx N_{\rm ew}\frac{2e^2\tau_{\rm w}\omega_{\rm pw}^2}{3c}\gamma_{\rm w}^4. \qquad (17)$$

The ratio of the energies of synchrotron radiation of all clusters (the number of clusters $n_{\rm cl}V$) in the focal volume $V = \pi\omega r_{\rm L}^4/c$ to the energy of betatron radiation of electrons of transparent plasma is expressed as

$$\mu = \frac{n_{\rm cl}V\varepsilon_\beta}{\varepsilon_{\rm w}} = \frac{3^4 n_{\rm cl}V\sqrt{Am_{\rm i}m_{\rm e}}\,c^2}{2^8 N_{\rm ew}\sqrt{Z}}\,\frac{\eta^3 a^{0.5}c^4\tau_{\rm L}^3}{R_0 e^2\omega r_{\rm L}^2}\,\frac{\omega_{\rm pw}^6}{\omega^6}. \qquad (18)$$

Note that the addition of clusters to the focal volume of a transparent plasma can change the energy of betatron radiation of electrons in this plasma. Thus, it was shown in [7] that clusters with a radius of several nanometres are (due to the Coulomb explosion) an additional source of electrons and additional scatterers of the electron beam, which increases estimate (17) by two to three orders of magnitude. In this work, we consider clusters with a radius of tens and hundreds of nanometres, the density of which is lower and for which there is no Coulomb explosion. A cluster plasma in the field of a circularly polarised laser pulse has an average longitudinal quasi-stationary magnetic field [3], which also affects the betatron radiation of an electron beam in a transparent plasma. It was shown in Ref. [16] that an external magnetic field opposite to the wave vector increases the energy of betatron radiation by tens of percent (approximately 1.5 – 2 times). In our case, the average magnetic field of the clusters is directed along the wave vector and should decrease the energy of betatron radiation. Thus, there are two opposite results of the effect of clusters on the betatron radiation of a homogeneous transparent plasma: on the one hand, clusters serve as an additional source of electrons, and on the other, their magnetic field suppresses betatron oscillations. We will focus on estimate (17) of the betatron radiation energy of electrons of a transparent plasma in order of magnitude, considering the effect of clusters with a radius of tens or hundreds of micrometres to be small or reduced to the appearance of a factor of the order of unity.

Let us estimate the parameter μ for real and numerical experiments and find the parameters of the cluster plasma and laser pulse for which $\mu > 1$. Yoo [19] used a laser intensity that was too low for clusters, 5×10^{18} W cm^{-2}, which leads to a low (tens of electronvolt) quantum energy of the synchrotron radiation, while the electrons accelerated by the plasma wave generate quanta with energies in the kilovolt range. In work [20], the dimensionless laser field amplitude $a = 100$ ($I = 10^{22}$ W cm^{-2}) exceeds the threshold value $a_{\rm th}$ corresponding to the Coulomb explosion for clusters with a radius of less than 100 nm. The most suitable laser parameters for our case were used in [21]: an intensity of 1.8×10^{19} W cm^{-2}, a pulse duration of 55 fs, and a beam radius of 16.4 μm. The beam propagates in a plasma with a subcritical electron density of 2.5×10^{18} cm^{-3}. The quantum energy of betatron radiation is 14.6 keV, the number of quanta is 1.2×10^9. For the same parameters of a laser pulse and one $\rm Au^{+30}$ cluster with a radius of 50 nm, according to the estimates of this work, we obtain a quantum energy of 4.5 keV and 10^{10} quanta of synchrotron radiation. At quantum energies above 14 keV, 3×10^8 quanta will be obtained from one cluster, but up to 10^3 clusters will fit in the caustic; therefore, the cluster plasma will be more efficient in terms of the yield of quanta with an energy of 14 keV. The conversion coefficient relative to the laser energy absorbed by the cluster for quanta with an energy of 4.5 keV is 4×10^{-2}, and for quanta with an energy of 14 keV, 1.3×10^{-3}.

Figure 3 shows the dependences of the ratio of energies μ on the laser intensity and the number of $\rm Au^{+30}$ clusters 50 nm in size, occupying the focal region of a laser pulse with a transverse radius of 4 μm (focal volume $\sim 2\pi^2 r_{\rm L}^4/\lambda \approx 5\times 10^3$ μm^3) and a duration $\tau_{\rm L} = 30$ fs. The maximum number of clusters that can fit in such a volume is $\sim 2\pi^2 r_{\rm L}^4/\lambda)/[4\pi(4R_0)^3/3] \approx 1.5\times 10^5$. In the experimental situation, the number of clusters is less than the possible maximum one; therefore, Fig. 3 shows estimates for 10^2 and 10^3 clusters in the focal volume and for two limiting values of the absorption coefficient, $\eta_0 = 0.3$ and 0.5 (in this case, the electron densities of the background plasma and clusters averaged over the focal volume are $\sim 10^{20}$ and 3×10^{20} cm^{-3}, respectively). It can be seen from Fig. 3 that with an increase in the number of clusters, the synchrotron radiation channel begins to prevail ($\mu > 1$). The threshold number of clusters in the focal volume at $\eta_0 = 0.3$ is $\sim 10^2$. Note that, in this case, the volume-averaged electron density is of the order of the background plasma density (clusters make a small contribution). However, with the dominance of the synchrotron channel (10^3 clusters), the average electron density exceeds the background one.

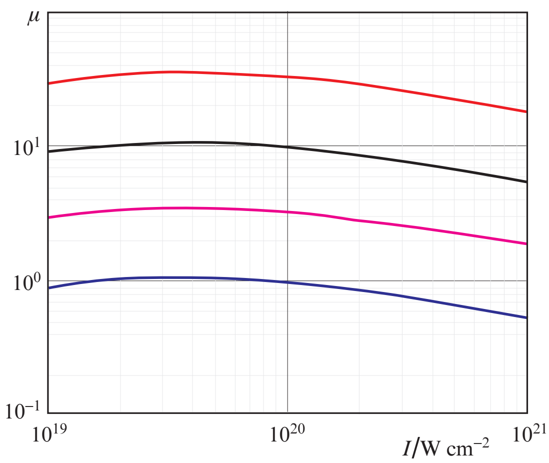

Figure 3. (Colour online) Dependences of the ratio μ of the energies of synchrotron and betatron radiation on the laser intensity and the number of $\rm Au^{+30}$ clusters with a radius of 50 nm, occupying the focal region of a laser pulse with a transverse radius of 4 μm and a duration of $\tau_{\rm L} = 30$ fs. The blue and lilac curves correspond to 10^2 clusters in the focal region and the limiting absorption coefficients are $\eta_0 = 0.3$ and 0.5, respectively; the black and red curves correspond to 10^3 clusters and $\eta_0 = 0.3$ and 0.5. The background plasma density is 10^{20} cm^{-3}. The average electron density (of the background plasma together with clusters) for 10^2 and 10^3 clusters in the focal volume is 10^{20} and 3×10^{20} cm^{-3}, respectively.

Thus, there are experimentally realisable ranges of the parameters of a gas cluster target and a laser pulse, for which the synchrotron channel of radiation of hard quanta is dominant in terms of the total radiation energy. However, it should be noted that the angular distributions of the energy of the betatron and synchrotron radiation are different. In the case of synchrotron radiation, the distribution represents a 'flat' disk, and in the case of betatron radiation, it is a narrow cone. Accordingly, the brightness of radiation (energy in a unit frequency range and in a unit solid angle) differ not by a factor of μ, but by a factor of $\sim\mu\gamma_{\rm w}\omega_{\rm pw}/(2\pi\omega_H\gamma^2)$ [the solid angles differ by a factor of $2\pi\gamma_{\rm w}^2/\gamma$, and the spectral intervals differ by a factor of $\omega_H\gamma^3/(\omega_{\rm pw}\gamma_{\rm w}^3)$], and the brightness of the betatron radiation can be higher than the brightness of the synchrotron radiation at a lower total energy. In Fig. 3, at an intensity $I = 10^{20}$ W cm^{-2}, we have the following parameters: $\gamma \approx 10, \gamma_{\rm w} \approx 60, \omega_{\rm pw}/\omega_H \approx 0.7$ and the factor $\gamma_{\rm w}\omega_{\rm pw}/(2\pi\omega_H\gamma^2)$ ≈ 0.1. Consequently, for the red curve in Fig. 3, the spectral-angular brightness of synchrotron radiation will be greater than the brightness of betatron radiation, and for the blue and lilac curves, it will be less. In the above estimates, the bremsstrahlung channel makes a small contribution in the range of quantum energies corresponding to the synchrotron and betatron radiation mechanisms. Only for quanta with energies $\varepsilon \gg \hbar\omega_{\rm pw}\gamma_{\rm w}^3$ and $e\hbar H_Q\gamma^2/(m_e c)$ the bremsstrahlung mechanism is dominant.

3. Conclusions

A cluster laser plasma produced by an intense circularly polarised laser pulse is a source of synchrotron and betatron secondary radiation. At low [up to $(3-5)\times10^{19}$ W cm^{-2}] laser intensities, the cluster plasma generates synchrotron quanta with low energy and cannot compete with a homogeneous subcritical density plasma in the efficiency of secondary radiation generation (conversion coefficient). However, in the range of intensities from 10^{20} to 10^{21} W cm^{-2} of circularly polarised laser radiation at optimal (50–200 nm) radii of heavy clusters (Au, Xe), the cluster plasma has a higher synchrotron radiation energy at comparable volumes in which radiation is generated (focal volume). The characteristic energies of synchrotron quanta (corresponding to the maximum of the radiation spectrum) in this range of laser intensities range from units to tens of keV (5–30 keV for Au clusters with a radius of 50 nm). A feature of synchrotron radiation of clusters is its angular distribution with a maximum in the direction transverse to the axis of the laser beam, in contrast to betatron radiation of a transparent plasma with a maximum in the longitudinal direction.

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