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A model for the formation of complex coupling coefficients in a ring resonator of a laser gyroscope

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Abstract. **A model of the interference of fields from backscattering point sources in a ring optical resonator is presented. Examples of calculations of complex coupling coefficients in ring resonators of laser gyroscopes are given. Comparison of the calculation results with the results of model experiments demonstrates good agreement and makes it possible to determine dissipative and conservative backscattering components of individual resonator mirrors. The possibilities of reducing the dissipative and conservative backscattering components in the process of ring resonator alignment are discussed.**

Keywords: *laser gyroscope, ring resonator, light backscattering, complex coupling coefficients, backscattering point sources, dissipative and conservative backscattering sources.*

1. Introduction

Backscattering caused by inhomogeneities of mirror coatings is one of the main sources of errors in a laser gyroscope (LG) based on a ring He–Ne laser with a wavelength $\lambda = 632.8$ nm. Of the three main parameters that characterise the LG accuracy – zero shift stability, angle random walk, and scale factor nonlinearity – the last two parameters are associated with backscattering. It is no coincidence that the improvement of technologies for polishing substrates and methods for applying multilayer dielectric mirrors has been a key factor that allows modern LGs to solve many precision problems of navigation, measuring angular displacements, geodesy, and geophysics.

In ring gas laser theory of [1–5], the effect of backscattering on the amplitude–frequency characteristics of an LG is described using two linear parameters, the so-called complex coupling coefficients (CCCs), which characterise parts of the field of eigenoscillations of a ring resonator that fall into a counterpropagating wave as a result of backscattering:

$$\boldsymbol{r}_{\mathrm{cw,\,ccw}} = r_{\mathrm{cw,\,ccw}} \exp(\mathrm{i}\varphi_{\mathrm{cw,\,ccw}}). \tag{1}$$

Here, the subscripts cw and ccw denote the parameters of the waves propagating along the ring resonator (RR) in the clockwise and counterclockwise directions, respectively; r_{cw} and r_{ccw} are the CCC moduli; and φ_{cw} and φ_{ccw} are phase shifts resulting from backscattering.

The total CCC is the result of the scalar summation of the partial components of all backscattering sources located in the working zones of the RR mirrors. This is true under the assumption that the selective diaphragm does not make a significant contribution to the CCC value.

If each of the resonator mirrors is represented as separate backscattering sources characterised by their own partial CCCs, then the total complex coupling coefficient is written in the form:

$$\boldsymbol{r}_{\mathrm{cw,ccw}} = \sum_n r_n \exp[\mathrm{i}(\pm 2kl_n + \varphi_n)], \tag{2}$$

where n is the serial number of the mirror; r_n is the modulus of the partial CCC; φ_n is the phase shift; l_n is the projection of the radius vector of the point source onto the optical axis of the RR (for brevity, we will call it the longitudinal coordinate); and $k = 2\pi/\lambda$ is the wavenumber. The presence of factor 2 in front of the wavenumber in formula (2) is due to the double phase incursion during the formation of the backscattering wave. The '+' sign refers to the wave in a clockwise direction.

The question immediately arises: What geometric dimension is meant by the longitudinal coordinates l_n? In the working area of the mirror, there are many defects randomly 'scattered' over its surface. How can one describe their position on the optical RR axis in the form of a longitudinal coordinate?

In the case of a point defect with a size much smaller than the wavelength, this problem does not seem to be difficult. The coordinate of a point backscattering source in the scalar sum (2) is the longitudinal coordinate of its position on the optical axis. When describing arrays of backscatteiring sources, the answer to the posed question is not so obvious. Moreover, under the conditions of real operation of the LG, the position of the optical axis of the ring resonator does not remain unchanged. Changes in the ambient temperature and atmospheric pressure lead to deformation of the optical contour and a shift of the laser mode along the surfaces of the RR mirrors.

It may seem strange, but the reason for writing this paper was an error in determining the longitudinal coordinate in relation (2), which we found in a paper published more than 30 years ago [6]. This could not have been reported. In understanding the physical nature of backscattering effects in a ring gas laser, there are still many 'blank spots' that are waiting for their researchers. However, references to work [6] continue to appear regularly in papers devoted to the LG problems. To date, the total citation list has amounted to 37 papers. This

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means that the error has gone unnoticed for developers and researchers of laser gyroscopes.

It should also be noted that when interpreting the results of measurements of the CCC moduli and the phase shift due to backscattering, we use a fundamentally different model of the CCC formation. The results of our model experiments on measuring CCCs in a ring resonator showed the correctness of using such a model. Before presenting the main results of our work, let us show what the error of the author of [6] consists of.

2. Model of point backscattering sources

The cause of the error in [6] lies in the fact that the longitudinal coordinate l_n in relation (2) is the length of the arm of the optical contour, at the vertices of which the RR mirrors are located. Figure 1a reproduces the optical scheme of a three-mirror RR used by the author of [6]. The resonator is formed by two flat mirrors and a spherical one. The spherical mirror is equipped with a piezoelectric transducer (PZT), which makes it possible to stabilise the resonator perimeter.

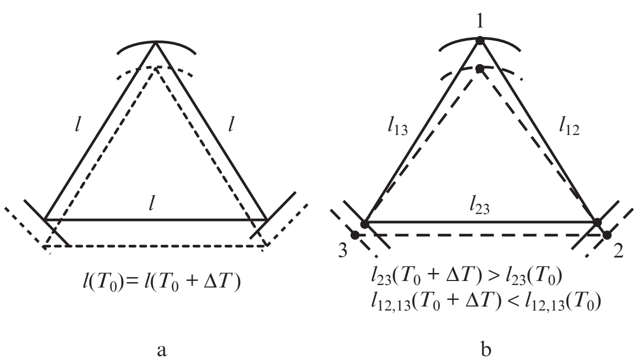

Figure 1. Formation of the total CCC in a three-mirror RR under uniform thermal deformations: (a) shift of the optical contour; (b) change in distances between point backscattering sources.

It is easy to verify that, under uniform thermal deformations, the shape of the resonator optical contour remains in the form of a regular triangle. The expansion of the RR housing is compensated by the shift of the PZT, which allows the lengths of the arms of the optical contour to remain unchanged. Hence, the author of [6] comes to the conclusion that when the PZT is installed on a spherical mirror, the modulus of the total CCC remains unchanged. The fallacy of this statement can be easily proven.

To do this, we concretise (and at the same time simplify) the model of backscattering sources. Let us assume that on the surface of each of the RR mirrors there is only one point source (Fig. 1b). In the initial position, these sources are located along the vertices of the optical contour. With the expansion of the RR housing, the distance between the point sources of flat mirrors increases, while the distances between the point sources of spherical and flat mirrors, on the contrary, decrease. The problem of calculating the modulus of the total CCC is reduced to calculating the distances between three point sources for various resonator deformations.

Let us start with the deformations that are associated with the PZT movement. When the PZT of a spherical mirror moves along the normal to a distance h, the RR perimeter changes by the value

$$\Delta L = h\sqrt{3}. \tag{3}$$

In this case, the distance between points 2 and 3 does not change, while the distances between points 1 and 2 and 1 and 3 decrease (for definiteness, we assume that the perimeter decreases when the PZT moves). These distances are defined as

$$l_{12} = l_{12}^{(0)} - h\frac{\sqrt{3}}{2}, \tag{4}$$

$$l_{13} = l_{13}^{(0)} - h\frac{\sqrt{3}}{2}, \tag{5}$$

where the superscript corresponds to the initial position of the mirrors.

Let us find out what will happen under uniform thermal deformations. Suppose we have increased the RR perimeter by ΔL. If the PZT is not used, then the distances between point sources will increase by $\Delta L/3$. The switching on of a stabilisation system compensates for this increase in perimeter. As a result, we obtain the relations for the distances between point sources:

$$l_{12} = l_{12}^{(0)} + \frac{\Delta L}{3} - h\frac{\sqrt{3}}{2}, \tag{6}$$

$$l_{13} = l_{13}^{(0)} + \frac{\Delta L}{3} - h\frac{\sqrt{3}}{2}, \tag{7}$$

$$l_{23} = l_{23}^{(0)} + \frac{\Delta L}{3}. \tag{8}$$

Let us now take into account the relationship between h and ΔL [see relation (3)]. This means that the change in the RR perimeter caused by the deformation is compensated by the movement of the PZT. The relations for the distances l_{12} and l_{13} take the form

$$l_{12} = l_{12}^{(0)} - \frac{\Delta L}{6}, \tag{9}$$

$$l_{13} = l_{13}^{(0)} - \frac{\Delta L}{6}. \tag{10}$$

Taking into account (6)–(10), we write relation (1) for the total CCC. In this case, we choose a spherical mirror as the reference point, and the rotation angles on the complex plane are measured from the partial CCC of this mirror. The total CCC can be written in the form:

$$\boldsymbol{r} = r_1 + r_2\exp\left(2ik\left(l_{12}^{(0)} - \frac{\Delta L}{6}\right)\right)$$
$$+ r_3\exp\left(2ik\left(l_{12}^{(0)} + l_{23}^{(0)} + \frac{\Delta L}{6}\right)\right). \tag{11}$$

The distances $l_{12}^{(0)}$ and $l_{23}^{(0)}$ between the point sources determine the rotation angles of the partial CCCs of the mirrors on the complex plane (see Fig. 1b). The vector r_1 remains immobile, since the initial position was chosen in this way. The vectors r_2 and r_3 rotate in different directions.

An example of the behaviour of the modulus of the total CCC under uniform thermal deformations is shown in Fig. 2. In the calculation, we used relation (11) written in the form:

$$\boldsymbol{r} = r_1 + r_2 \exp\Big(2\mathrm{i}k\Big(a_2 - \frac{\Delta L}{6}\Big)\Big)$$

$$+ r_3 \exp\Big(2\mathrm{i}k\Big(a_3 + \frac{\Delta L}{6}\Big)\Big). \quad (12)$$

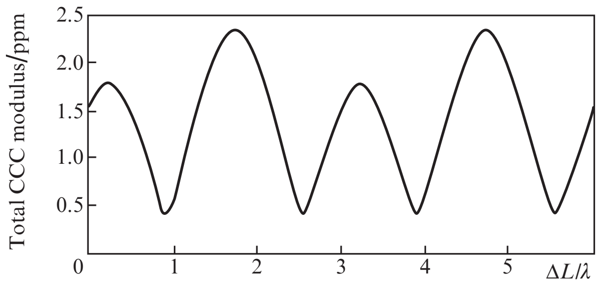

Figure 2. Dependence of the total CCC modulus on the relative change in the perimeter under uniform thermal deformations of a three-mirror RR. Initial data: r_1 = 0.5 ppm, r_2 = 1 ppm, a_2 = 0.3, r_3 = 1 ppm, and a_3 = 0.1.

As can be seen, we have obtained a periodic two-hump dependence $r(\Delta L)$ with a period of 3λ. With such a change in the perimeter, the distances between neighbouring point sources change by 1λ, which corresponds to a 360° rotation of the vectors of partial CCSs on the complex plane.

Let us give one more example illustrating the incorrectness of determining the longitudinal distance between the backscattering sources in the form of arms of the optical contour of the RR. Let us assume that a flat mirror is shifted along the contact surface of the RR monoblock housing. In this case, the optical contour of the resonator does not change. Let us see what happens with the CCC modulus when the mirror moves over a distance comparable to the wavelength.

Note that in the case of the mirror moving in the direction perpendicular to the plane of the optical contour, the longitudinal distances between the point sources of the RR mirrors practically do not change. For example, for a ring resonator arm length (distance between adjacent mirrors) l = 70 mm, a shift from the centre of a point defect by δ = 10 μm leads to an increase in the distance Δl_n by 1/1400 μm ($\Delta l_n \approx \delta^2/2l$), i.e., the modulus of the total CCC remains virtually unchanged.

The situation dramatically changes when the mirror is moved in the plane of the optical contour (Fig. 3). In this case, when the mirror moves a distance δ, the relations for the distances between point sources take the form

$$l_{12} = l_{12}^{(0)} + \frac{\delta}{2}, \quad (13)$$

$$l_{13} = l_{13}^{(0)} - \frac{\delta}{2}. \quad (14)$$

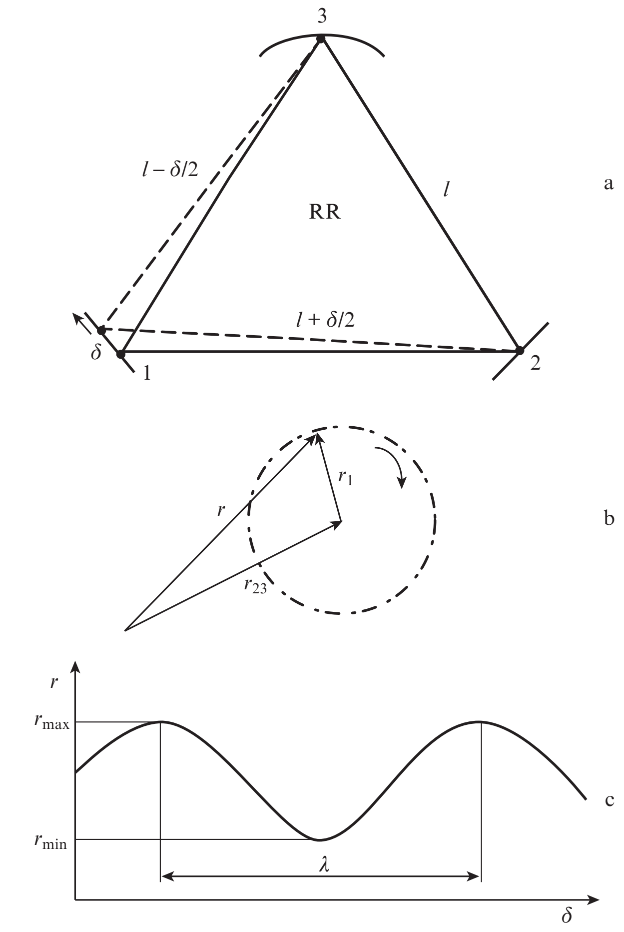

Figure 3. (a) Geometry of mirror movement along the contact surface of the RR housing, which allows controlling the phase of the mirror's partial CCC; (b) formation of the total CCC on the complex plane; and (c) dependence of the total CCC modulus on the longitudinal displacement of the mirror.

As follows from (13) and (14), when mirror 1 moves along the edge of the monoblock housing, the distances l_{12} and l_{13} change antiphase: one of them increases and the other decreases. The value of the total CCC is described by the relation:

$$\boldsymbol{r} = r_2 + r_3 \exp(2\mathrm{i}k l_{23}^{(0)})$$

$$+ r_1 \exp\Big[2\mathrm{i}k\Big(l_{12}^{(0)} + l_{13}^{(0)} - \frac{\delta}{2}\Big)\Big]. \quad (15)$$

As applied to the CCC vectors on the complex plane, this means that the vector r_1 rotates about the sum vector r_{23}. For $\delta = \lambda$, the change in the distances l_{12} and l_{13} is $\lambda/2$, i.e., the vector r_1 rotates through 360°. The value of δ is the period of dependence of the total CCCF modulus on the mirror displacement.

Figure 3 shows the corresponding dependence of $r(\delta)$ for the total CCCS modulus. Based on its maximum (r_{max}) and minimum (r_{min}) values, we can express the ratio for the partial contributions r_1 and r_{23} in the form:

$$r_{1,23} = \frac{r_{max} \pm r_{min}}{2}. \quad (16)$$

At first glance, there is something wrong with this formula. On the left-hand side there are two indices, and on the right-hand side in the numerator there is a sign ±. And it is not clear how they are related. However, this is a completely correct form of presentation. The fact is that the total CCC is the result of the interference of two backscattering sources (in this case, these are r_1 and r_{23}), and so it is impossible to determine which their CCC modulus is greater than the other. This is a typical uncertainty in the analysis of an interference pattern formed by two sources. Relation (16) correctly illustrates this uncertainty. Below we will return to the problem of identification and show how, in a number of cases, it is possible to identify the contribution of the backscattering source.

Let us now consider the optical scheme of a ring resonator, in which uniform deformations do not lead to displacement of the optical contour vertices on the mirror surfaces (Fig. 4). We have a four-mirror RR with two spherical mirrors installed diagonally across the optical contour and equipped with a PZT. Under uniform expansion, the optical contour of the resonator turns from a square into a rhombus. The distance between the point sources of mirrors 2 and 4 increases, and the distance between the point sources of mirrors 1 and 3 decreases. The lengths of the arms of the optical contour (they are also the distances between neighbouring point defects), as well as the rotation angles of partial CCCs, do not change. Therefore, the modulus of the total KKS also remains unchanged.

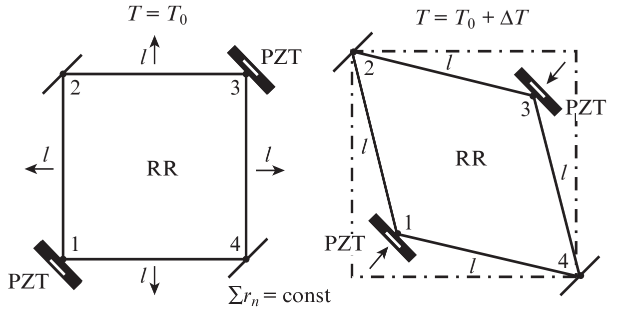

Figure 4. Deformation of the optical contour during uniform expansion of a ring four-mirror resonator, in which the PZTs are mounted on two spherical mirrors.

When the RR perimeter changes by 1λ, the length of each arm of the optical circuit changes by $\lambda/4$, which corresponds to a 180° rotation of the complex variables of the partial CCCs, and the value of the modulus of the total CCF changes. When changing the perimeter to 2λ, we obtain the original geometry of the vectors.

The above-considered particular examples of the formation of the total CCC made it possible not only to illustrate the error in determining l_n in the form of the lengths of the arms of the optical contour. Based on this consideration, we can formulate the main provisions of the model for the formation of the total CCC in a ring resonator.

First of all, when the optical contour is deformed, it is necessary to monitor the movements of the geometric centres of the mirrors. Changes in the total CCC are described by relation (2), in which the longitudinal coordinates of the geometric centres of the mirrors on the optical RR axis are treated as the parameters l_n. The position of any of the resonator mirrors can be taken as the initial reference point.

Another important assumption on which our model is based is the invariance of the partial contribution of each of the mirrors to the total CCC under deformations of the optical resonator contour. We assume here that the optical contour deformations, at which only the longitudinal components l_n change in relation (2), are small. The moduli of the partial CCCs do not change in this case.

The designs of modern LGs, whose monoblock housings are made of materials with ultra-low thermal expansion coefficients (for example, zerodur or glass-ceramic), make it possible to avoid noticeable deformations of the optical contour even under 'hard' conditions of their use (for example, in the operating temperature range of ± 50 °C). In this case, the angle of inclination of the optical axis, as a rule, does not exceed several arcseconds, and the transverse displacement of the axis does not exceed several tens of micrometres.

To conclude this section, we make the following important remark. We have considered the simplest case, when the surfaces of the RR mirrors contain one point defect each. The model can be made more complex. Let us imagine a mirror as an array of point sources randomly 'scattered' over its surface. In this case, the CCC of the mirror is represented as a scalar sum (2), where the values of the parameters r_n, φ_n and l_n characterise the point sources located on its surface. This means that we again come to the model of one 'effective' point source, whose parameters are determined by the sum of all point sources.

Next, we consider how this model works when measuring the CCC in real monoblock ring resonators.

3. Model of point backscattering sources in describing experiments on measuring CCCs

3.1. Measurement of the partial CCC moduli of individual mirrors of a ring resonator using a scheme with a return mirror

The use of the method for measuring the CCC in RRs not filled with an active gas mixture [7] makes it possible to check the correctness (or incorrectness) of the model for the formation of backscattered fields. To this end, a model experiment was set up with a four-mirror ring resonator.

In the experiment, use was made of a resonator with a perimeter of 16 cm and mirrors coated with $Ta_2O_5-SiO_2$. The total integral scattering (TIS) of the mirrors was 20–30 ppm. PZTs were installed on two resonator mirrors. The hermetic housing of the RR was filled with air and had a glass appendix, the internal volume of which was comparable to the total volume of the internal cavities of the RR (Fig. 5a). The presence of the appendix allowed the length of the optical contour arms to be changed in a controlled manner. For this purpose, the appendix was heated with a nichrome coil. At the same time, the air density inside the monoblock housing increased, the temperature of which remained at room temperature.

When measuring the CCC modulus, a scheme with a returnable mirror (RM) was used. The technical details of the measurement method can be found in [7]. Here we confine ourselves to a brief description.

An eigenmode oscillation is excited in the RR being measured using single-mode radiation from a probe laser. The use of a frequency stabilisation unit (FSU) makes it possible to 'lock-in' the laser generation frequency to the RR natural oscillation frequency. The radiation emerging from the RR is incident on the RM and returns back, exciting its eigenoscil-

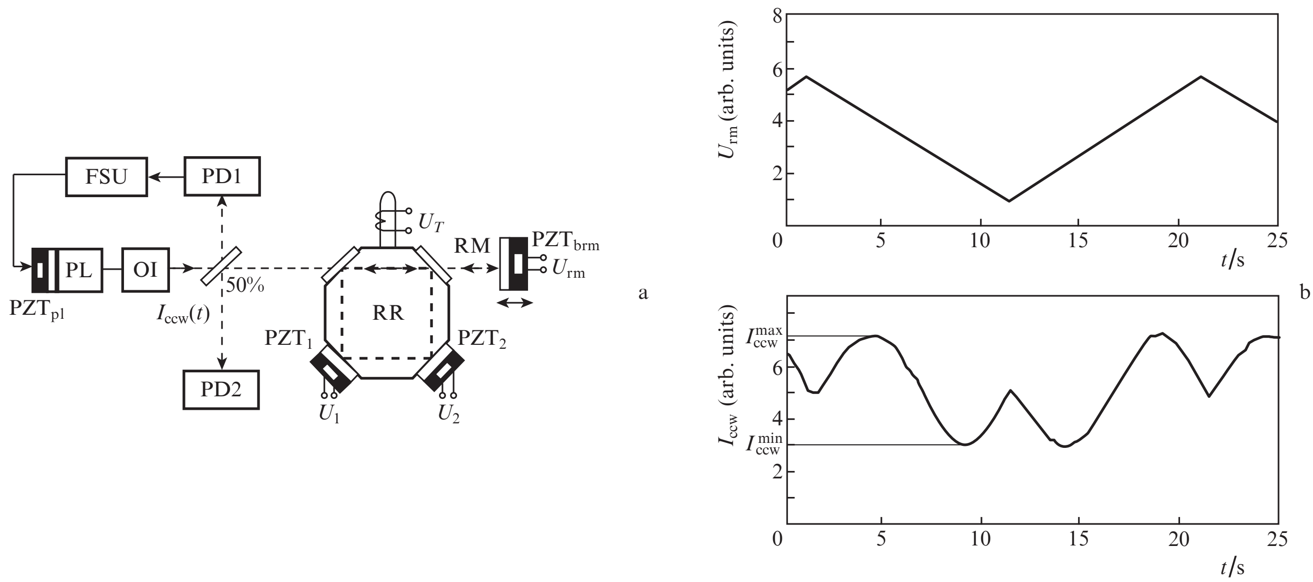

Figure 5. (a) Scheme of a model experiment for measuring partial CCCs of RR mirrors [(PL) probe He–Ne laser; (FSU) frequency stabilisation unit; (OI) optical isolator; (50%) semitransparent plate; (PD1) and (PD2) photodetectors; (PZT_{pl}, PZT_1, PZT_2, PZT_{brm}) piezoelectric transducers; (RR) ring resonator with an appendix; (RM) returnable mirror; U_T is the voltage for controlling the heating temperature T of the appendix of the ring resonator], as well as (b) time dependences of the voltage applied to the returnable mirror and the intensity of radiation emerging from the RR.

lation in the opposite direction. With longitudinal displacement l of the returnable mirror, an alternation of maxima and minima with a period equal to $\lambda/2$ is observed in the intensity of the counterpropagating wave (I_{ccw}) emerging from the RR (Fig. 5b). These changes are the result of the interference of the eigenoscillations of the resonator with backscattered waves. The intensity of the radiation emerging from the RR is described by the relation:

$$I_{ccw} = A\left[\frac{T}{F} + r_{cw}\cos(2kl)\right]^2, \quad (17)$$

where F is the attenuation coefficient (in intensity) of the filter installed in front of the RM (we assume that its reflection coefficient is 100%); T is the transmittance (in intensity) of the output mirror; and A is a constant coefficient depending on the transmittance of the input and output mirrors, as well as on the RR losses.

Relation (17) means that two CCCs are added on the complex plane. In the case when $T/F \gg r_{cw}$, it takes the form:

$$I_{ccw} \approx A\left(\frac{T}{F}\right)^2\left[1 + 2\frac{r_{cw}F}{T}\cos(2kl)\right]. \quad (18)$$

This dependence is characterised by the contrast value, which includes the minimum (I_{ccw}^{min}) and maximum (I_{ccw}^{max}) intensity values recorded when moving the RM:

$$C_{ccw} = \frac{I_{ccw}^{max} - I_{ccw}^{min}}{I_{ccw}^{max} + I_{ccw}^{min}} = \frac{2r_{cw}F}{T}. \quad (19)$$

The contrast value can be increased in a controlled manner by increasing the attenuation factor of the filter. This simplifies the registration of intensity extrema and the determination of C_{ccw}. Given the pre-measured values of F and T, we determine the CCC modulus r_{cw}. In this example, there is no uncertainty in identifying the backscattering sources: the wave reflected from the RM, as a rule, significantly exceeds the intensity of the backscattered wave.

To determine the partial contributions of the mirrors, we use the regime of antiphase movement of two PZTs mounted on adjacent mirrors of the ring resonator. With such a displacement of the PZT, the RR perimeter does not change. One PZT increases the perimeter, while the other compensates for this increase. The measured dependence of $r(U_{PZT})$, where U_{PZT} is the voltage of one of the PZTs, is a periodic function of the voltage with a period corresponding to a change in the perimeter by 1λ (Fig. 6a).

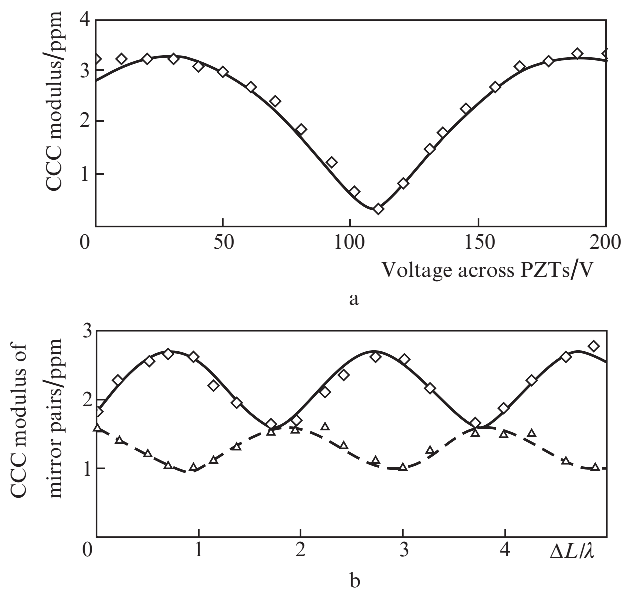

Figure 6. Results of measurements of the partial CCC moduli: (a) dependence of the CCC modulus on the voltage across the PZTs during their antiphase motion and (b) dependence of the CCC modulus of pairs of mirrors on the relative change in the RR perimeter.

With such a displacement of the piezoelectric transducers, the total CCC modulus is the result of the interference of two scattering sources: pairs of mirrors 1–2 and 3–4. The regime of antiphase displacement of two PZTs makes it possible to determine the partial CCC moduli for mirror pairs. Thus the partial moduli are the half-sum and half-difference of the maximum and minimum values of the total CCC modulus:

$$r_{12,34} = \frac{r^{\max} \pm r^{\min}}{2}. \tag{20}$$

Thus, we obtain a pair of unidentified values of the CCC moduli for pairs of mirrors.

After that, the resonator appendix begins to warm up, and the distance between the mirrors increases. With a smooth change in this distance, the reference point changes and another pair of values, r_{12} and r_{34}, is obtained. Due to the fact that the dependences of $r_{12}(\Delta L)$ and $r_{34}(\Delta L)$, where ΔL is the change in the perimeter during the heating of the appendix, are smooth and periodic with a period of 2λ, it is possible to 'link' these values with each other. As a result, we have two continuous dependences (Fig. 6b). As expected, these are periodic dependences on the change in the RR perimeter with a period equal to 2λ. With such a change in the perimeter, each resonator arm changes its length by $\lambda/2$, and the partial CCCs on the complex plane rotate by 360°.

Having measured the maximum and minimum values of $r_{12}(L)$ and $r_{34}(L)$, we determine the partial values of the CCC moduli. In this case, with a relative error of $\sim$20%, they were: $r_1 = 0.3$ ppm, $r_2 = 1.3$ ppm, $r_3 = 0.5$ ppm, and $r_4 = 2.1$ ppm. Obviously, the uncertainty in the index numbers implies the possibility of changing the pair of mirrors 1–2 to the pair 3–4, as well as changing the indices within the pair. Nevertheless, we succeeded in determining the partial contributions to the total CCC of all the mirrors of RR in question.

Note that when assembling this RR, we used mirrors that had approximately the same quality. The value of the total integral scattering (TIS) of the mirrors was about 30 ppm. Such a large scatter in the values of the CCC moduli is mainly due to the speckle structure of the backscattering field inside the RR [8]. In this case, the parameter distribution histograms characterising the backscattering of the mirror arrays of the ring resonator obey the Rayleigh distribution [9]

$$f(\rho) = \frac{\rho}{\sigma^2}\exp\left(-\frac{\rho^2}{2\sigma^2}\right), \tag{21}$$

where $f(\rho)$ is the distribution density of the random parameter ρ; and σ is the scale parameter, approximately equal to the average value of the parameter ρ.

A characteristic property of this distribution is that the average value of the parameter coincides with the value of its root-mean-square deviation (RMS) obtained by processing the data array. In the considered example, the average value of the CCC modulus for four mirrors was 1.05 ppm at RMS = 0.8 ppm. Of course, such an insignificant array of data is clearly not enough to generalise the results; however, there is no obvious contradiction in the obtained measurement results.

3.2. Model of point backscattering sources in the description of CCCs of counterpropagating waves

Until now, the process of the formation of the CCC modulus of one of the counterpropagating RR waves has been described by the model of point backscattering sources o. This greatly simplifies the analysis of the situation. We limited ourselves to consideration of relation (2) for the CCC modulus, which includes constant values of the partial CCC moduli and linear displacements of the mirrors. For the same reason, phase shifts arising during backscattering on mirrors do not enter into this relation. Formally, they can be included as additions to the longitudinal coordinates of point sources, especially since the position of a point source does not necessarily coincide with the geometric centre of the mirror.

It is easy to show that when all backscattering sources have the same phase shift ($\varphi_n = \varphi_{\rm cw} = \varphi_{\rm ccw}$), the CCC moduli of the counterpropagating waves are the same ($r_{\rm cw} = r_{\rm ccw}$). The situation changes qualitatively when the backscattering sources included in relation (2) have different phase shifts. Let us show this by the example of the interference of two point backscattering sources (Fig. 7).

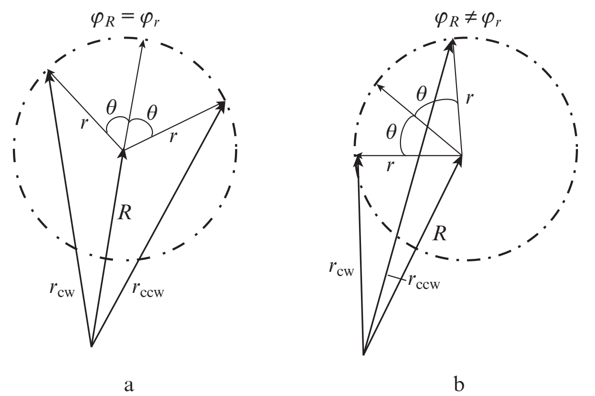

Figure 7. Vector diagrams of the summation of two CCCs of point backscattering sources in the case of (a) identical ($\varphi_R = \varphi_r$) and (b) different ($\varphi_R \neq \varphi_r$) phase shifts.

Each of these sources is characterised by its partial CCC modulus, the value of phase shift that occurs during backscattering, and the longitudinal coordinate on the optical RR axis. The total CCC of this system can be written in the form:

$$\boldsymbol{r}_{\rm cw,ccw} = R\exp({\rm i}\varphi_R) + r\exp({\rm i}\varphi_r \pm 2{\rm i}kl). \tag{22}$$

When the difference between their longitudinal coordinates changes, a difference appears between the CCC moduli of counterpropagating waves, which is described by the relation:

$$r^2_{\rm cw} - r^2_{\rm ccw} = 4rR\sin(2kl)\sin(\varphi_R - \varphi_r). \tag{23}$$

As regards phase shifts of backscattered waves, one should take into account the structure of the system of equations describing the amplitudes of counterpropagating waves and their phase difference (see, for example, [1]). The phase shifts during backscattering enter these equations as a sum

$\varphi = \varphi_{cw} + \varphi_{ccw}$. As a result, the equations do not contain an addition to the phase shift $2kl$ associated with the longitudinal coordinate of the source, and the total value of the phase shift turns out to be invariant over the entire optical contour of the RR, independent of the reference point on the optical axis. Therefore, by the CCC of counterpropagating waves, one should mean not four parameters (two moduli and two phase shifts), but three, i.e. r_{cw}, r_{ccw}, and φ.

Returning to the interference of the backscattered fields of two point sources (with different values of the phase shift), we note that when the difference in their longitudinal coordinates changes, the total phase shift does not remain constant and changes in a wide range from 0 to 2π. Relations (22) and (23) involve two point sources with arbitrary values of the phase shift caused by backscattering. To consider the model for the formation of the CCC, we specify these values.

The authors of Refs [1, 3] mention two types of backscattering sources: conservative and dissipative. In the case of a conservative backscattering source, the phase shift is π/2 (or π for the total phase shift). This type of backscattering is associated with inhomogeneities in the refractive index. Dissipative backscattering is caused by an absorption coefficient that is inhomogeneous over the mirror surface, and the magnitude of the phase shift in this case is π (2π or 0 for the total phase shift). Nonlinear distortions of the LG scale factor are determined by the influence of both types of backscattering. In particular, in an LG without a bias, the dependence of the nonlinear correction of the scale factor on the angular velocity of rotation, $\Delta K(\Omega)$, is described by the well-known relation [10]

$$\Delta K = \frac{\Delta\nu}{\Omega} - 1 = -\left(\frac{c}{L}\right)^2\left[\frac{(S_+)^2}{2\Omega^2} - \frac{1}{2}\frac{(S_-)^2}{\Omega_g^2 + \Omega^2}\right], \tag{24}$$

where c is the speed of light; $\Delta\nu$ is the frequency of beats of counterpropagating LG waves; and Ω_g is the limit cycle strength of the ring laser. Parameters S_+ and S_- are the following CCC combinations:

$$S_+ = \sqrt{r_{cw}^2 + r_{ccw}^2 + 2r_{cw}r_{ccw}\cos\varphi}, \tag{25}$$

$$S_- = \sqrt{r_{cw}^2 + r_{ccw}^2 - 2r_{cw}r_{ccw}\cos\varphi}\,. \tag{26}$$

The first combination S_+ is a dissipative backscattering component and determines the lock-in threshold (Ω_l) of the LG:

$$\Omega_l = \frac{c}{L}S_+. \tag{27}$$

The second combination S_- is a conservative backscattering component and determines the positive sign of the correction of the LG scale factor at high angular velocities of rotation.

The experiments performed in [10, 11] showed that in an LG based on a 632.8-nm ring He–Ne laser, there is no correlation between the backscattering components, and the average value of the conservative component is 5–7 times higher than the average value of the dissipative component. Therefore, when describing the model for the formation of a CCC of a mirror, we assumed that there are two arrays of point backscattering sources, a conservative and a dissipative one, on its surface. Consequently, in relations (22) and (23) we can assume that $\varphi_R = \pi/2$ and $\varphi_r = \pi$.

Within the framework of this model, three parameters are used in the formation of the CCC of the mirror: the CCC moduli of the conservative (R) and dissipative (r) backscattering components, as well as the phase shift parameter $\zeta = 2kl$, where l is the difference between the longitudinal coordinates of point sources. The CCC moduli of the backscattering components and ζ are related to the CCC moduli r_{cw}, r_{ccw}, and φ as follows:

$$R = \frac{S_-}{2}, \tag{28}$$

$$r = \frac{S_+}{2}, \tag{29}$$

$$\zeta = \arcsin\left[\frac{r_{cw}^2 - r_{ccw}^2}{\sqrt{\left(r_{cw}^2 + r_{ccw}^2\right)^2 - 4r_{cw}^2 r_{ccw}^2\cos^2\varphi}}\right]. \tag{30}$$

We used these relations to determine the CCC moduli of the backscattering components from the results of measurements of the CCCs in four-mirror RRs [7].

When measuring the CCC of counterpropagating waves, a scheme with an optical mixer [7] (Fig. 8a) is used, in which eigenoscillations in opposite directions are simultaneously excited in the measured RR by the radiation of a probe laser. When the mixer mirrors are moved in the intensities of the waves (I_{cw} and I_{ccw}) emerging from the RR, small (on the level of fractions of a percent) intensity variations are observed, which are associated with the interference of the fields of natural oscillations and backscattered waves. The dependences of the intensities I_{cw} and I_{ccw} on the phase difference χ of the excited eigenoscillations are described by the relations:

$$I_{cw} \approx E_0^2\left[1 + \frac{4r_{ccw}}{\delta}\cos(\chi + \varphi_{ccw})\right], \tag{31}$$

$$I_{ccw} \approx E_0^2\left[1 + \frac{4r_{cw}}{\delta}\cos(\chi - \varphi_{cw})\right], \tag{32}$$

where E_0 is the field amplitude. The process of measuring the CCC is based on the introduction of a controlled change in the phase difference χ. The contrast of the intensities of the radiations emerging from the RR is directly proportional to the moduli of the corresponding CCCs, r_{cw} and r_{ccw}, and the shift between their extrema is determined by the total phase shift $\varphi = \varphi_{cw} + \varphi_{ccw}$ that occurs during backscattering.

Of greatest interest in this case are the dependences of the CCC moduli obtained with antiphase movement of two PZTs. In this case, it is possible to determine the CCC moduli of conservative and dissipative backscattering components for two pairs of mirrors. The result of one of these measurements is shown in Fig. 8b. The values of r_{cw}, r_{ccw} and φ were measured experimentally.

The measured dependences of the CCC moduli and the phase shift were approximated using relations (28)–(30). The results of this approximation are shown in Fig. 8b by solid lines. The calculated dependences of the CCC moduli of the conservative and dissipative backscattering components are

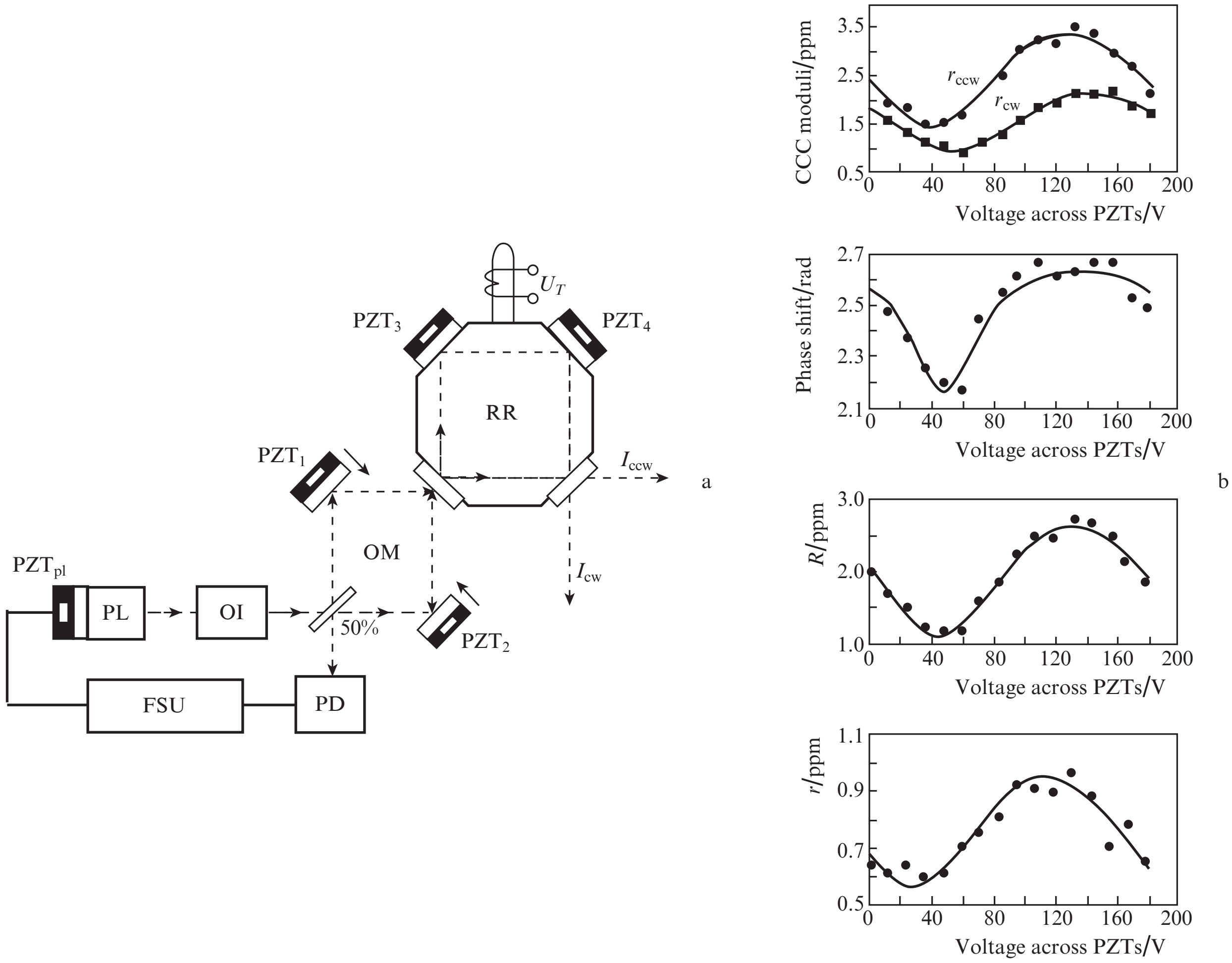

Figure 8. (a) Optical scheme using an optical mixer for measuring the CCC moduli of counterpropagating waves in a four-mirror RR with antiphase movement of two PZTs [(PL) probe He–Ne laser; (OI) optical isolator; (OM) optical mixer; (PZT_{pl}, PZT_1–PZT_4) piezoelectric transducers; (RR) ring resonator; (FSU) frequency stabilisation unit; (50%) semitransparent plate; (PD) photodetector; U_T is the voltage for controlling the heating temperature T of the appendix of the ring resonator) and (b) results of CCC measurement.

also presented in the figure. One can see that the used model of interference of the fields of conservative and dissipative backscattering sources describes well the results of our experiments. Within the framework of this model, it is possible to explain the noticeable difference in the CCC moduli of counterpropagating waves, r_{cw} and r_{ccw}, observed in a number of experiments, as well as the behaviour of the total phase shift φ during the antiphase movement of the PZTs.

In addition, a model experiment was performed with a RR having a glass appendix. In this case, we used a four-mirror RR with a perimeter of 28 cm; the mirrors were coated with Ta_2O_5–SiO_2. The TIS of the mirrors was about 20 ppm. The resonator was formed by two flat mirrors and two spherical ones with curvature radii of ~6 m. Two PZTs were installed on neighbouring spherical mirrors. The cyclogram of above-described measurements was reproduced when determining the partial CCC moduli of the mirrors. This made it possible to measure for each of the RR mirrors the CCC moduli of the dissipative and conservative backscattering components. The results of measurements of the CCC moduli of the dissipative components are presented in Table 1.

During the experiment, we performed two cycles of measurements. After the first cycle, four partial values of the dissipative components were determined. Then, one of the flat mirrors of the resonator was removed from the monoblock housing and, before being reinstalled on the optical contact, it was turned around its axis by an angle of ~90°. After that, the second cycle of measurements of the dissipative components was carried out and four more values were determined.

Table 1. Partial CCC moduli of the dissipative backscattering components of the mirrors of a four-mirror resonator.

Mirror number	Dissipative component (1st measurement cycle)/ppm	Dissipative component (after mirror rotation)/ppm
1	0.38	0.1
2	0.24	0.17
3	0.15	0.11
4	0.1	0.09

In Table 1, the obtained values are grouped as follows. Mirrors 2, 3, and 4 are grouped according to the approximate equality of values in two measurement cycles. The partial CCCs of these mirrors did not change when one flat mirror was reinstalled. The difference in the values of the dissipative components in two measurement cycles is due to the error of the measuring setup. The relative error in this case did not exceed 20%. One of the values (mirror 1) changed when the mirror was rotated almost 4 times. Such a large difference in the measured values makes it possible to identify this mirror.

One more of the mirrors can also be identified. Let us show that this is a flat mirror 2. The fact is that with this method of measurement (antiphase movement of the PZT), we determine the CCCF values for two pairs of mirrors – movable (on the PZT) and stationary. Having identified one of the mirrors, we have identified a pair. This means that mirror 2 was identified by the elimination method. This is a flat mirror. For a pair of spherical mirrors, it is not possible to identify the number of mirrors in this case.

As in the case of measurements of partial CCC moduli, the scatter of the CCC moduli of the dissipative backscattering components turned out to be significant. It is noteworthy that these values for mirror 1 changed by a factor of 4 when it was rotated around its axis. This behaviour of the CCC is a direct consequence of the action of the speckle structure of the backscattered fields in the ring resonator. This feature of the behaviour of partial CCCs when the mirror is rotated can be used to lower the LG lock-in threshold during alignment.

The use of arrays of dissipative and conservative backscattering sources allows the adjustment process to be simulated. In this case, the main adjustable parameter is the angle of rotation of the mirror around its axis. The authors of Ref. [11] drew attention to this feature of the behaviour of backscattering speckle structures. They suggested turning the RR mirror in such a way that the dark field of the speckle structure of the radiation scattered by the mirror would fall into the counterpropagating wave. The simulation results of such an adjustment process are presented below.

4. Model of a mirror consisting of two independent arrays of dissipative and conservative point sources

Let us consider a model of two independent arrays of point backscattering sources (dissipative and conservative) randomly scattered over the mirror surface. Each of these arrays includes 1000 sources with the same value of the CCC modulus. The measurement data of the CCCs in the RR [7, 10] for multilayer dielectric mirrors with $Ta_2O_5-SiO_2$ and TiO_2-SiO_2 layers show that the conservative component is 3–7 times larger than the dissipative component. In our calculations, we set the value of the ratio of these components equal to 5.

Let us assume that all point sources have the same CCC moduli: R_0 are conservative and r_0 are dissipative. In this case, the total values of the CCC moduli of the arrays are presented in the form:

$$R = R_0 \left| \sum_{n=1}^{1000} \exp(\mathrm{i}\phi_n) \right|, \tag{33}$$

$$r = r_0 \left| \sum_{m=1}^{1000} \exp(\mathrm{i}F_m) \right|, \tag{34}$$

where ϕ_n and F_m are quantities that vary randomly in the range from 0 to 2π. When performing calculations, we set the values of the moduli of point sources as follows: $R_0 = 0.05$ ppm and $r_0 = 0.01$ ppm.

Using a random number generator (1000 pairs of ϕ_n and F_m values), it is possible to obtain histograms of the distributions of the CCC moduli of the conservative and dissipative backscattering components. With 10^6 realisations, the shape of these histograms differs little from the Rayleigh distribution (Fig. 9).

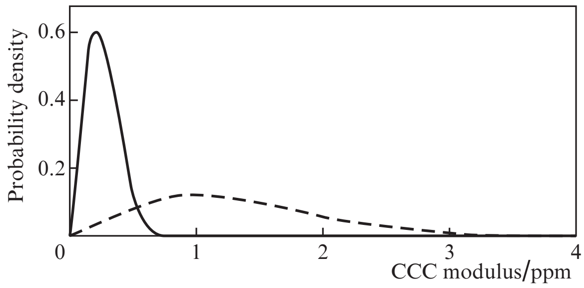

Figure 9. Rayleigh distributions of the CCC moduli for (solid curve) dissipative and (dashed curve) conservative components of arrays consisting of 1000 point backscattering sources; $R_0 = 0.05$ ppm and $r_0 = 0.01$ ppm.

In the case when all backscattering sources have the same phase shift value, the modulus of the total CCC is the sum of the moduli of individual point sources. With a random value of the phase shift, the average value of the CCC modulus is approximately 2% of this value: 1 ppm for conservative sources and 0.2 ppm for dissipative ones. A similar behaviour of the histograms of the conservative and dissipative backscattering components was observed in [10], where statistical data are presented for 220 LGs, collected using mirrors of approximately the same quality. At the same time, there was a lack of correlation between the measured values of these two backscattering components.

Let us consider how the CCC moduli will change when the mirror rotates around its axis. The mirror is located on the edge of the monoblock housing of the LG, and so the angle of incidence remains unchanged (we will assume the mirror being adjusted to be flat). When the mirror is rotated, the coordinates of point sources on the optical RR axis change (Fig. 10). As a result, the phase shifts ϕ_n and F_m also change. Of course, the change in the phase of a point source is determined mainly by its position relative to the rotation axis. Let us try to estimate the scale of the change in the phase shift. We assume that the point source is located from the mirror rotation axis at a distance equal to the waist radius w of the RR mode. When the mirror is rotated by an angle Φ, the phase will change by $4\pi w\cos\Phi/\lambda$ (we assume that $\Phi \ll 1$).

Let us perform numerical estimates for $w = 300$ μm and $\lambda = 0.63$ μm. At such values, the phase incursion of the backscattering source will change by the value π when the mirror is rotated through an angle $\Phi \approx 0.0005$ rad (approximately 10″). In the case of arrays of backscattering sources, randomly scattered on the working zone the mirror, the phase shifts ϕ_n and F_m in relations (33) and (34) will change in different ways; for example, for a backscattering source located on the rotation axis, the value of the phase shift will remain unchanged.

It is clear that the type of dependence of the CCCs of arrays of point sources on the angle of rotation of the mirror is determined by their initial distribution on its surface. As an

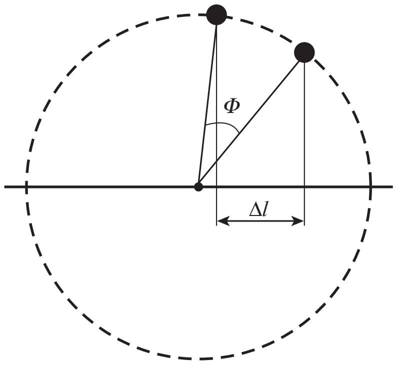

Figure 10. Change in the phase of a point backscattering source when the mirror rotates around its axis. The horizontal axis coincides with the projection of the optical contour of the resonator.

example, Fig. 11 shows the dependences of the CCC moduli r_{cw} and r_{ccw} on the angle of rotation, as well as the total phase shift for two arrays of point sources. The initial position of 1000 point sources of arrays is chosen using a random number generator.

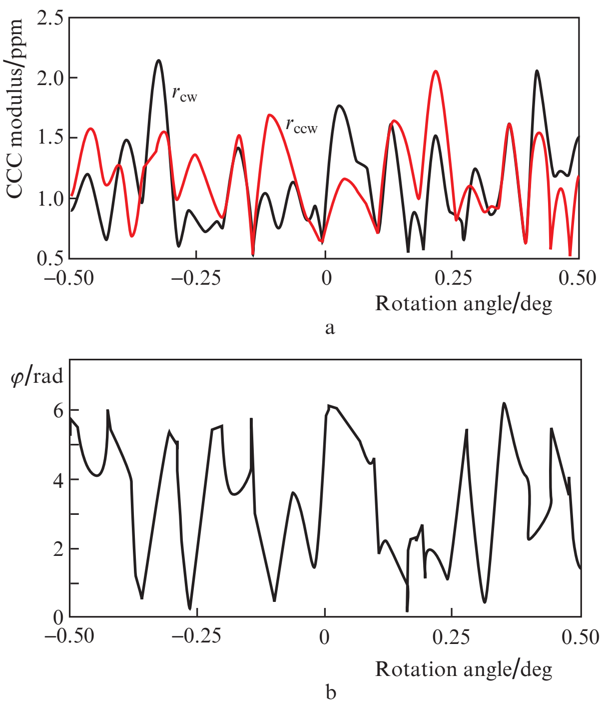

Figure 11. (Colour online) Dependences of (a) the CCC moduli and (b) total phase shift $\varphi = \varphi_{cw} + \varphi_{ccw}$ on the angle of rotation of the mirror for two arrays of 1000 point backscattering sources; R_0 = 0.05 ppm and r_0 = 0.01 ppm.

The resulting dependences are quite complex. Analysing them, we can distinguish the following three features. The first of them is associated with a large spread in the values of the CCC moduli. If we consider the rotation angle Φ as a random variable, then the histograms of the chosen conservative and dissipative backscattering components of the arrays the form of a Rayleigh distribution. For this reason, Fig. 11 shows only a small fragment of these dependences when the rotation angle changes by ±0.5 deg. It can be seen that the characteristic angular scale of the change in the CCC moduli is 1–2 arc minutes. The second feature is due to the difference in the CCC moduli for counterpropagating waves, which at some angles of rotation of the mirror can reach 10 times. The third feature is the change in the value of the total phase shift in a wide range of values 0–2π. Note that all three features in the behaviour of the backscattering parameters are observed in CCC measurements in LGs [7], i.e., the proposed model of point backscattering sources is in qualitative agreement with the experimental results.

From a practical point of view, the influence of the angle of rotation of the mirror on the values of the moduli of the conservative and dissipative components can be used to organise the RR alignment process. As can be seen from the above dependences presented, at some values of the angle of rotation, it is possible to achieve a noticeable decrease in the values of the CCC moduli of both backscattering components.

5. Conclusions

The model of point backscattering sources adequately describes the results of CCC measurements in the LG. This model is based on two main assumptions. Firstly, it is assumed that the partial CCCs of the mirrors do not change with deformations of the contour of the optical RR axis, and secondly, that the phase shifts in the scalar sum (2) are determined by the distances between the geometric centres of the mirrors.

The partial CCcs of an individual mirror are represented as a scalar sum of two arrays of point sources, conservative and dissipative, differing in the magnitude of the phase shift that occurs during backscattering. Taking into account the fact that there is no correlation in the positions of these arrays on the mirror surface [10], the backscattering of the mirror is characterised by three parameters. These include the CCC moduli of the conservative and dissipative backscattering components, as well as the phase shift associated with a change in the positions of the arrays relative to each other.

The number of point defects in the working area of the LG can be estimated as 10^6–10^7 or more. This estimate follows from the fact that the surface roughness of the mirror substrate is characterised by a correlation length of tenths of a micrometre. Such a large number of defects, randomly 'scattered' over the mirror surface, is the cause of the speckle structure of the scattered laser radiation. The backscattered field is a complex combination of bright and dark spots and stripes. The angular size of an individual speckle is several arc minutes ($\lambda/2w$, where $2w$ is the mode waist diameter). The same value is also given by the estimate of the RR aperture [12]. For this reason, no more than one bright or dark speckle spot of scattered radiation enters the RR aperture. This leads to a large scatter of the CCC moduli of the backscattering components of ring resonators assembled from mirrors with the same value of the total integral scattering. It is no coincidence that the histograms of the distributions of these parameters (for a batch of several hundred RRs) are well described by the Rayleigh distribution.

This feature of the behaviour of the backscattered fields can be used in the development of an alignment method that makes it possible to significantly reduce the CCC moduli of the dissipative and conservative backscattering components. When the mirror rotates around its axis by an angle equal to fractions of a degree, the CCC moduli of the conservative and dissipative backscattering components change by more than an order of magnitude. To measure the partial CCC moduli of the conservative and dissipative backscattering components, it is necessary to move the mirror being adjusted along the contact surface of the monoblock housing (in the plane of the optical contour) by a distance of $\sim 1\lambda$. The partial CCC contribution of the mirror being aligned is determined from the analysis of variations in radiation intensities of counter-propagating waves emerging from the RR.

It is obvious that the authors are aware of all the difficulties associated with the use of the method of measuring the CCCs when adjusting LG resonators. The main difficulty lies in solving the technical problems that arise during the movements of the aligned mirror, which consist of its rotation around its axis and movement along the contact surface of the monoblock housing. These movements should not be accompanied by noticeable perturbations of the RR perimeter, which lead to a failure of the frequency stabilisation unit that locks in the generation frequency of the probe laser to the frequency of the natural oscillation of the aligned resonator. Also undesirable are changes in the angle of incidence of radiation on the mirror by more than $1''$, leading to a noticeable shift of the optical axis of the RR relative to the position of the selective diaphragm.

Thus, when designing such an alignment setup, the central task is to create a device for moving the mirror being adjusted. One may also have to correct the characteristics of the frequency stabilisation unit. At present, when measuring the CCC, the frequency of the radiation of a probe laser is stabilised by resonances of the power of the radiation emerging from the RR. An error signal formed from the first derivative of the resonance is applied to the PID controller with a cutoff frequency of about 200 Hz. To create an alignment setup, one may need a faster FSU.

To date, installations for measuring the CCC have an interesting metrological option that allows one to control the backscattering parameters at all technological stages of assembly and vacuum-technological processing of the RR in an LG. In particular, we are able to determine the CCC moduli of the dissipative and conservative backscattering components for individual resonator mirrors, predict the values of nonlinear corrections of the LG scale factor and the value of the lock-in threshold. We hope that the results of this work will attract the attention of developers and help reduce the impact of backscattering effects in LGs.

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