Quantum Electronics **52** (4) 402–406 (2022)

https://doi.org/10.1070/QEL18020

Application of PT-symmetry properties to increase the limiting sensitivity of resonator optical gyroscopes

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Abstract. **We propose and describe the use of a system with PT-symmetry properties in resonator optical gyroscopes. It is shown that this makes it possible to increase the limiting sensitivity and does not introduce disadvantages typical of PT-symmetric laser gyroscopes. In the example considered in the paper, the inclusion of a system with PT-symmetry properties in the composition of a resonator optical gyroscope allows one to increase the limiting sensitivity by a factor of 750.**

Keywords: *optical gyroscope, angular velocity sensor, passive ring cavity, Sagnac effect, PT symmetry.*

1. Introduction

PT-symmetric (Parity-Time symmetry) systems are a poorly studied class of non-Hermitian systems with unique properties. They are characterised by a PT-symmetric Hamiltonian, i.e., their Hamiltonian commutes with the operators of spatial inversion P and time reversal T [1]:

$$[\hat{H}, \hat{P}\hat{T}] = \hat{P}\hat{T}\hat{H} - \hat{H}\hat{P}\hat{T} = 0. \tag{1}$$

Since the beginning of the 21st century, interest in such systems has been continuously growing. This is explained by the expansion of the quantum-mechanical concept of PT-symmetric systems, from which research on this topic began, to optics, where the practical implementation of such systems is currently possible. In recent years, several reviews have been published on non-Hermitian optics and the prospects for its development in both fundamental and applied research [1–3]. In optical systems, the PT-symmetry condition of the Hamiltonian in (1) imposes a constraint on the permittivity of a medium (or, equivalently, on the refractive index): The real part of the permittivity must be symmetric, $\mathrm{Re}[\varepsilon(f, x, y, z)] = \mathrm{Re}[\varepsilon(f, -x, -y, -z)]$, and the imaginary part must change the sign, $\mathrm{Im}[\varepsilon(f, x, y, z)] = -\mathrm{Im}[\varepsilon(f, -x, -y, -z)]$ (f is the frequency of optical radiation; and x, y and z are the axes of the Cartesian coordinate system) [3]. In other words, the losses in one part of the system ($\mathrm{Im}[\varepsilon] > 0$) must be equal to the gain ($\mathrm{Im}[\varepsilon] < 0$) in the opposite part (Fig. 1).

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Received 6 November 2021; revision received 26 January 2022
Kvantovaya Elektronika **52** (4) 402–406 (2022)
Translated by I.A. Ulitkin

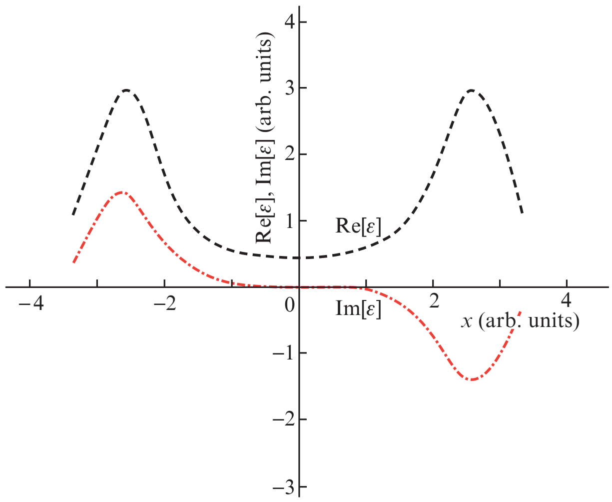

Figure 1. (Colour online) Example of the distribution along the x axis (at $y = z = 0$) of the permittivity of a medium in a PT-symmetric system.

In such systems, several modes can simultaneously exist. At small and equal values of loss and gain, the system is in a PT-symmetric phase, and with gradually increasing loss and gain in a medium and reaching a certain value (determined by the coupling between the modes), a singularity (phase transition point) is achieved. Beyond this point, spontaneous violation (destruction) of the PT symmetry occurs and real eigenvalues of the system Hamiltonian undergo transition to complex eigenvalues [1]. In this case, one part of the modes begins to be absorbed, while the other part is sharply enhanced. Thus, a phase transition, which can be directly or indirectly caused by a change in the measured physical quantity, leads to a sharp change in the optical properties of the system. This can be used to improve the accuracy of various optical sensors, in particular angular velocity sensors.

To date, several papers have been published on the measurement of angular velocity using PT-symmetry systems [4–7]. The authors of Refs [4–7] consider various variations of laser gyroscopes based on systems of two identical coupled ring resonators, which differ from each other only in the level of loss and gain. At the same time, they report that such gyroscopes can be more accurate than classical laser gyroscopes in the region of low angular velocities and, theoretically, they have no lock-in zones [4]. However, they have a nonlinear output characteristic, reduced sensitivity to high angular velocities and do not allow one to determine the direction of rotation. The PT-symmetry system used in these gyroscopes must be located at a singularity, which imposes severe restrictions on the gain, loss, and coupling coefficient of the resonators. In addition, not only rotation, but also other external and internal factors lead to a displacement of the system from a singularity and are perceived by such sensors as rotation. For example, even small ($\sim 10^{-3}$ °C) fluctuations in the resonator temperature make it almost impossible to measure small angular velocities [5]. Thus, these disadvantages outweigh the listed advantages. Note that most of these disadvantages are due to the use of an active sensitive element, which is part of the PT-symmetry system. In this regard, in our work, we consider the application of the PT-symmetry properties in gyroscopes with passive resonators (in resonator optical gyroscopes). In this case, the simplest system of two coupled waveguides, which differ from each other only in loss and gain levels, is used as a system with PT-symmetry properties.

2. System of two waveguides with PT-symmetry properties

To increase the sensitivity of resonator optical gyroscopes, we propose to use one of the simplest optical PT-symmetry systems implemented in practice. It is an optical integrated circuit and is formed by two straight parallel waveguides 1 and 2 (Fig. 2a). Waveguides 1 and 2 are located at a sufficient distance from each other to maintain optical coupling between their modes due to the optical tunnelling effect. The value of this coupling can be given by the waveguide coupling coefficient g. The permittivity of this system is distributed similarly to Fig. 1: The real part of the permittivity is symmetrical, and the imaginary part changes the sign when the x axis passes through zero. Thus, waveguide 1 is characterised by a gain constant γ_1, and waveguide 2 is characterised by a loss constant γ_2. In all other resppects the waveguides are identical.

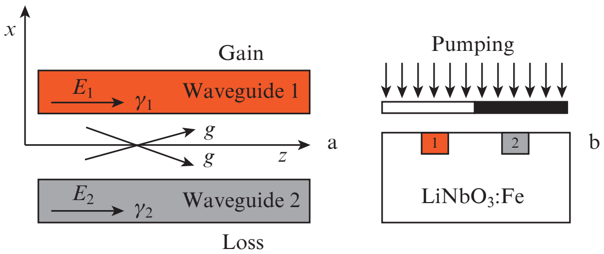

Figure 2. (Colour online) Optical system with PT symmetry: (a) signal propagation in the system (E_1 and E_2 are the radiation field strengths); (b) an example of the implementation of pumping.

The gain and losses in the waveguides are maintained by optical pumping supplied from above or below the optical integrated circuit (Fig. 2b). The practical feasibility of the described system has already been proven. Previously, similar PT-symmetry systems have already been fabricated, for example, based on lithium niobate doped with iron [8] or aluminium gallium arsenide [9].

The distributions of electric fields in waveguides 1 and 2 can be represented as follows: $E_{1,2}(x, y, z) = U_{1,2}(z)V_{1,2}(x, y)$, where $U_{1,2}(z)$ are distributions of field amplitudes along the z axis, and $V_{1,2}(x, y)$ are the distribution of field amplitudes in the cross section [9]. In this study, of interest are only the functions $U_{1,2}(z)$, which can be defined in terms of the Hamiltonian of the system [1]:

$$\mathrm{i}\frac{\partial}{\partial z}\begin{bmatrix} U_1(z) \\ U_2(z) \end{bmatrix} = \begin{bmatrix} H_{11} & H_{12} \\ H_{21} & H_{22} \end{bmatrix}\begin{bmatrix} U_1(z) \\ U_2(z) \end{bmatrix}$$

$$= \begin{bmatrix} \beta_0 + \mathrm{i}\gamma_1 & g \\ g^* & \beta_0 - \mathrm{i}\gamma_2 \end{bmatrix}\begin{bmatrix} U_1(z) \\ U_2(z) \end{bmatrix}, \quad (2)$$

where $[H]$ is the Hamiltonian matrix of the system; β_0 is the propagation constant of radiation for one isolated waveguide without taking into account losses and amplification (i.e., in the case of its removal from the system); γ_1 and γ_2 are constant gains and losses of waveguides 1 and 2, respectively; and g is the complex coupling coefficient of the waveguides. The system of differential equations (2) is reduced to a linear homogeneous differential equation of the second order:

$$U_1''(z) + \mathrm{i}(H_{11} + H_{22})U_1'(z)$$

$$+ (H_{12}H_{21} - H_{11}H_{22})U_1(z) = 0. \quad (3)$$

The roots of the characteristic equation corresponding to expression (3) have the form

$$o_{\pm} = \frac{\mathrm{i}}{2}\left[-H_{11} - H_{22} \pm \sqrt{4H_{12}H_{21} + (H_{11} - H_{22})^2}\right]. \quad (4)$$

Based on Eqns (2)–(4), we can determine the distributions of the fields $U_{1,2}(z)$ formed by two modes (+ and –) propagating in the system consisting of waveguides 1 and 2:

$$U_1(z) = C_+\mathrm{e}^{o_+ z} + C_-\mathrm{e}^{o_- z},$$

$$U_2(z) = C_+\left(\frac{\mathrm{i}o_+ - H_{11}}{H_{12}}\right)\mathrm{e}^{o_+ z} + C_-\left(\frac{\mathrm{i}o_- - H_{11}}{H_{12}}\right)\mathrm{e}^{o_- z}, \quad (5)$$

where $C_{\pm}$ are constants determined from the initial conditions. Using expressions (2), (4), and (5), we can determine the propagation constants of these two modes:

$$\beta_{\pm} = \mathrm{i}o_{\pm} = \frac{1}{2}\left[H_{11} + H_{22} \mp \sqrt{4H_{12}H_{21} + (H_{11} - H_{22})^2}\right]$$

$$= \beta_0 + \frac{\mathrm{i}}{2}(\gamma_1 - \gamma_2) \mp \sqrt{|g|^2 - \frac{1}{4}(\gamma_1 + \gamma_2)^2}. \quad (6)$$

Let us now analyse how a change in the loss in waveguide 2 affects the amplification/attenuation of the + and – modes. For greater clarity, we will use (6) to construct the dependences of the propagation constants of modes $\beta_{\pm}$ on γ_2 (Fig. 3). When constructing dependences in Fig. 3, the value of $|g|$ was set equal to 3.28 cm^{-1}.

For $\gamma_1 = \gamma_2$ (Fig. 3a), the described system can be called PT symmetric, i.e., the relations $\mathrm{Re}[\varepsilon(f, x, y, z)] = \mathrm{Re}[\varepsilon(f, -x, -y, -z)]$ and $\mathrm{Im}[\varepsilon(f, x, y, z)] = -\mathrm{Im}[\varepsilon(f, -x, -y, -z)]$ are fulfilled. For $\gamma_1 = \gamma_2 < |g|$ the system is in a PT-symmetric phase. In this case, its modes do not experience gain and loss. For $\gamma_1 = \gamma_2 > |g|$ the system will undergo transition into a phase with broken PT symmetry, and the + and – modes will experience loss and gaim, respectively. It should be noted that it is very difficult to realise and maintain a strict balance between the gain and loss of waveguides 1 and 2 ($\gamma_1 = \gamma_2$) in practice due to

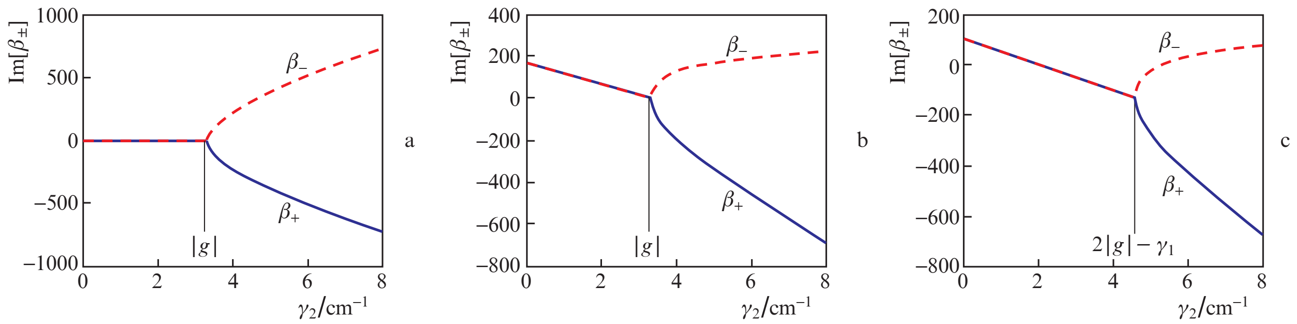

Figure 3. (Colour online) Dependences of the mode propagation constants $\beta_{\pm}$ on γ_2 for (a) $\gamma_1 = \gamma_2$, (b) $\gamma_1 = |g|$, and (c) $\gamma_1 = 0.6|g|$.

the instability of the environment and optical pumping, errors in the manufacture of waveguides, etc. Therefore, from a practical point of view, of greater interest are systems that do not have a strict balance of gain and loss ($\gamma_1 \neq \gamma_2$), but have the properties of PT-symmetric systems (Figs 3b and 3c). They do not satisfy the condition $\mathrm{Im}[\varepsilon(f, x, y, z)] = -\mathrm{Im}[\varepsilon(f, -x, -y, -z)]$, but there is a 'hidden' phase transition, and through formal transformations the systems can be reduced to PT-symmetric ones [1, 3]. Below we will call such systems "systems with PT-symmetry properties". The phase transition point of such systems corresponds to the condition $\gamma_2 = 2|g| - \gamma_1$, i.e., with increasing loss in waveguide 2, there also occurs a phase transition, after which a sharp change in the attenuation and amplification of the + and – mode fields is observed. This event can be recorded with high accuracy, which makes it possible to use the system data to improve the accuracy of measuring various physical quantities.

3. Application of the PT-symmetry properties in resonator optical gyroscopes

Various types of ring resonators are used as sensitive elements of resonator optical gyroscopes [10]. The principle of their operation is based on the measurement of the angular velocity Ω by the Sagnac-effect induced splitting of eigenfrequencies of the passive ring resonator, Δf_{S}. This splitting is proportional to the angular velocity

$$\Delta f_{\mathrm{S}} = f_{\mathrm{cw}} - f_{\mathrm{ccw}} = \frac{4A}{\lambda L}\Omega, \tag{7}$$

where f_{cw} and f_{ccw} are the eigenfrequencies of the resonator for waves travelling around it clockwise and counterclockwise, respectively; A is the area covered by the resonator contour; $\lambda = c/f_0$; c is the speed of light in vacuum; f_0 is the eigenfrequency of an immobile resonator; and L is the resonator perimeter. Figure 4 shows a generalised scheme of a resonator optical gyroscope. The radiation from a tunable laser (*1*) is split by a U-shaped coupler (*2*) into two channels. The wave frequencies in these channels are regulated by passing through phase modulators (*3*, *4*). Then, using a directional coupler (*9*), the radiation is launched into a passive ring resonator (*10*) in both clockwise and counterclockwise directions. Through directional couplers (*7*, *8*, *9*), light enters photodetectors (*5*, *6*). Signals from photodetectors are sent to a computer system (*11*). By changing the amplitude characteristic of the passive ring resonator recorded by photodetectors (*5*, *6*), the computer system (*11*) determines the eigenfrequencies f_{cw} and f_{ccw} and using phase modulators (*3*, *4*) and/or the tunable laser (*1*) adjusts the wave frequencies for them in two channels of the gyroscope. The angular velocity is determined by the system from the eigenfrequency difference, $f_{\mathrm{ccw}} - f_{\mathrm{cw}}$, in accordance with expression (7). The principle of operation of resonator optical gyroscopes is described in more detail in [10, 11]. The limiting sensitivity of gyroscopes of this type to the angular velocity $\delta\Omega$, i.e., the minimum change in the angular velocity that can potentially be measured against the background of photon noise of photodetectors (quantum measurement limit), is determined by the expression [12]

$$\delta\Omega = \frac{cL\sqrt{2}}{4Af_0}\sqrt{\frac{hf_0\Delta\nu}{I_{\mathrm{in}}}}\left[\max\left(\frac{1}{\sqrt{I_{\mathrm{n}}}}\frac{\mathrm{d}I_{\mathrm{n}}}{\mathrm{d}f}\right)\right]^{-1}, \tag{8}$$

where h is Planck's constant; $\Delta\nu$ is the bandwidth of the signal processing system; I_{in} is the power at the input of the optical circuit of the gyroscope; $I_{\mathrm{n}} = I_{\mathrm{out}}/I_{\mathrm{in}}$ is the normalised power at the output of the optical circuit of the gyroscope (transmission coefficient); and I_{out} is the power at the output of the optical circuit of the gyroscope (the power sent to the photodetector).

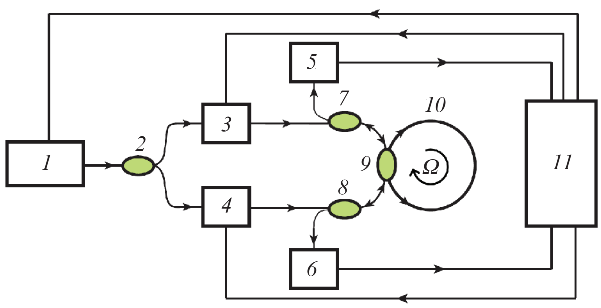

Figure 4. Generalised scheme of a resonator optical gyroscope: (*1*) tunable laser; (*2*) coupler; (*3*, *4*) phase modulators; (*5*, *6*) photodetectors; (*7*, *8*, *9*) directional couplers; (*10*) passive ring resonator; (*11*) computing system.

As noted above, due to the phase transition, PT-symmetric systems can drastically change the optical properties, which is proposed to be used to increase the steepness of the transmission coefficient of resonator optical gyroscopes, $\mathrm{d}I_{\mathrm{n}}/\mathrm{d}f$. It can be seen from (8) that this should reduce the minimum measurable change in the angular velocity (i.e., increase the limiting sensitivity). To implement this, it is possible to connect the system with the PT-symmetry properties, described in Section 2, to a passive ring resonator (Fig. 5).

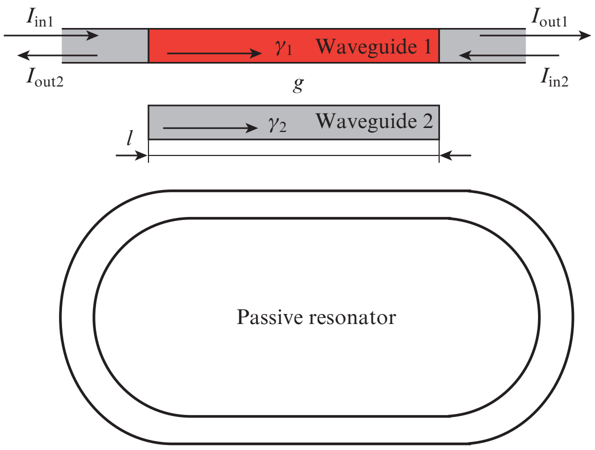

Figure 5. (Colour online) System with PT-symmetry properties connected to a passive ring resonator.

We will assume that waveguides 1 and 2 (Fig. 5) are identical, except for the gain and loss in them. For simplicity, we will assume that the resonator connected to the system with PT-symmetry properties has the shape of a track and is optically coupled only to waveguide 2 throughout the entire length l of the system.

It is known that the ratio of the electric field strengths of radiation at the output and input of a passive ring resonator is determined by the expression [10]

$$t = \frac{y - xe^{-i\delta_p}}{1 - xye^{-i\delta_p}}, \tag{9}$$

where $x = \sqrt{1-P}$; $y = \sqrt{1-K}$; $\delta_p = 2\pi LN(f - 0.5\Delta f_S)/c$ is the phase incursion acquired by the wave in one round trip in the resonator; P is the fraction of energy lost by the wave in one round trip in the resonator; K is the energy coupling coefficient of the resonator with a passive waveguide; and N is the effective refractive index of the resonator waveguide. Connecting a passive ring resonator to the described system with PT-symmetry properties can be described by adding the term $H_p = i\ln t/l$ to the H_{22} element of the system's Hamiltonian.

Using (2), (4), (5) and (9), one can determine the transmission coefficients for a passive ring resonator, I_{n0}, and a resonator connected to a system with PT-symmetry properties, $I_{n1} = I_{out1}/I_{in1}$, as well as their slope: dI_{n0}/df and dI_{n1}/df, respectively (Fig. 6). The following system parameters were used when plotting the dependences in Fig. 6: $P = K = 0.1$, $L = 6$ cm, $N = 2.48$, $|g| = 3.28$ cm^{-1}, $\gamma_1 = 0.3|g|$, $\gamma_2 = 0.9|g|$, $l = 1$ cm, $\lambda = 1.5$ μm. The listed values are achievable in practice. When choosing the values of the gain and loss constants, the following factors were taken into account: fulfilment of the condition $\gamma_2 < 2|g| - \gamma_1$ (i.e., without taking into account the passive ring resonator, the system must be in a symmetrical phase); the maximum value of the transmission coefficient I_{n1} should not exceed two. The selected values of the parameters, although not optimal, make it possible to visually evaluate the advantages of the idea proposed in this paper. The simulation also took into account the width of the laser emission line equal to 20 kHz. Such a width corresponds to the characteristics of tunable single-frequency semiconductor lasers commonly used in resonator optical gyroscopes [13].

One can see from Fig. 6 that the transmission coefficient I_{n0} for the passive ring resonator decreases as the eigenfre-

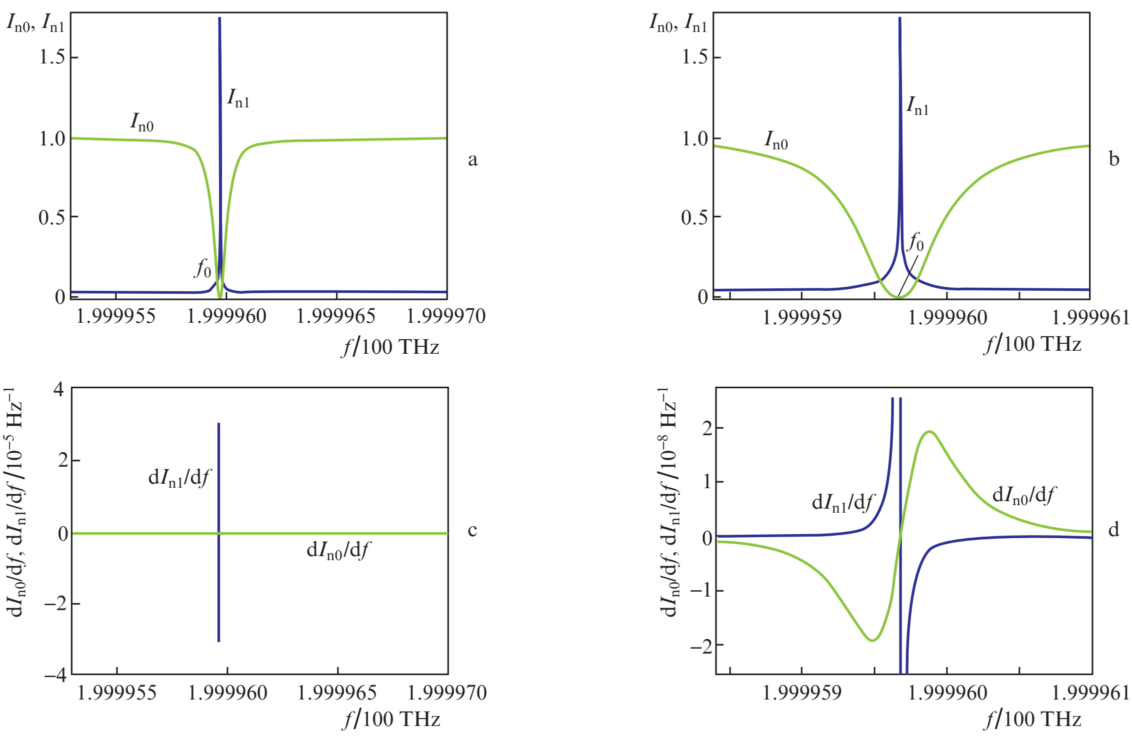

Figure 6. (Colour online) (a) Transmission coefficients for a passive ring resonator, I_{n0}, and a resonator connected to a system with PT-symmetry properties, I_{n1}; (b) section of dependences in Fig. 6a on a scale increased in frequency; (c) frequency dependences of the steepness of the transmission coefficients; and (d) section of dependences in Fig. 6c on a scale increased in frequency.

quency is approached. As a result, connecting a passive ring resonator to a system with PT-symmetry properties leads to an additional increase in the loss of the passive waveguide 2 by an amount depending on the light frequency f. Thus, when approaching eigenfrequencies, the loss in a system with PT-symmetry properties increases, it passes the phase transition point, and this causes a sharp change in the transmission coefficient I_{n1} (Fig. 6). As can be seen from Figs 6c and 6d, the use of a system with PT-symmetry properties makes it possible to increase the maximum value of the transmission coefficient slope by several orders of magnitude. In accordance with (8), the limiting sensitivity of the resonator optical gyroscope is the higher, the greater the value of

$$\max\left(\frac{1}{\sqrt{I_n}}\frac{dI_n}{df}\right).$$

With these parameters, connecting a system with PT-symmetry properties increases the ultimate sensitivity of the resonator optical gyroscope by approximately 750 times. For example, at $\Delta\nu = 1$ kHz and $I_{in} = 1$ mW, the limiting sensitivity of a conventional resonator gyroscope will be 141 deg h^{-1}, and when using the proposed system, it will increase to 0.188 deg h^{-1}.

In addition, the considered approach is devoid of the disadvantages inherent in PT-symmetric laser gyroscopes. As follows from (7), the resonator optical gyroscope has a linear output characteristic, the same sensitivity over the entire range of measured angular velocities, and allows the direction of rotation to be measured. With the considered approach to measuring the angular velocity, there are no strict requirements on the values and stability of the parameters of a system with PT-symmetry properties. A change in the parameters $|g|$, γ_1, and γ_2 does not affect the splitting of the eigenfrequencies of the passive ring resonator determined in resonator optical gyroscopes, from which the angular velocity is calculated [see (7)]. However, their instability can affect the ultimate sensitivity. With the above gyroscope parameters, a decrease in γ_1 by 10% will worsen the limiting sensitivity by only 8%, and a change in $|g|$ by 10% will worsen the limiting sensitivity by 10%. Increasing γ_1 or $|g|$ by 10%, on the contrary, will increase the maximum sensitivity by 9%. A similar change in γ_2 has an even weaker effect on the sensitivity. It is worth noting that there are such values of the parameters $|g|$, γ_1, γ_2, P, and K, at which the maximum value of the transmission coefficient increases to 100 or higher. In practice, this can lead to saturation of the gain of the active waveguide medium and to an increase in spontaneous emission in a system with PT-symmetry properties, which will adversely affect the gyroscope operation. This does not prevent the fabrication of a gyroscope, but additionally complicates the task of optimising the system parameters.

Thus, the use of the above-described system with the PT-symmetry properties in a resonator optical gyroscope makes it possible to increase its limiting sensitivity and does not introduce negative factors characteristic of PT-symmetric laser gyroscopes.

4. Conclusions

The paper briefly considers the concept of PT-symmetry and the conditions necessary to design optical PT-symmetric systems. The phase transition observed in optical PT-symmetric systems can be used to improve the accuracy of various optical sensors, including angular velocity sensors. To date, several studies have been published on such sensors. They consider various variations of PT-symmetric laser gyroscopes. However, PT-symmetric laser gyroscopes are of little interest from a practical point of view due to a significant number of critical drawbacks: stringent requirements on the values and stability of parameters of a PT-symmetric system, sensitivity of these parameters to various external and internal factors (for example, temperature fluctuations), and nonlinearity of the output characteristic and uncertainty of the rotation direction.

The paper proposes to use systems with PT-symmetry properties in resonator optical gyroscopes. It is shown that this makes it possible to increase the limiting sensitivity and does not introduce the negative factors characteristic of PT-symmetric laser gyroscopes. In this case, one of the simplest systems, which has already been implemented in practice more than once, is used as a system with PT-symmetry properties. It is an optical integrated circuit and is formed by two straight parallel optically coupled waveguides, differing from each other only in the level of gain and loss. In the example considered in the paper, its inclusion in the composition of a resonator optical gyroscope makes it possible to increase the limiting sensitivity by a factor of 750.

Acknowledgements. The work was supported by the Ministry of Science and Higher Education of the Russian Federation (Grant No. FSEE-2020-0005).

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