Two-level gas laser with transverse diode pumping

A.I. Parkhomenko, A.M. Shalagin

Abstract. We theoretically study a new method for generating laser radiation by a two-level system without population inversion in the 'red' wing of its spectral line under resonant absorption of broadband radiation from pump laser diodes. A two-level system simulates the atoms of an active gas in an atmosphere of a high-pressure buffer gas. The effect results from the fact that in the 'red' wing of the spectral line, the probability of stimulated emission exceeds the probability of absorption if the homogeneous broadening due to the interaction of particles with the buffer gas significantly exceeds the natural one (at high pressures of the buffer gas). Analytical formulae are obtained that describe the operation of a two-level gas laser with transverse diode pumping. It is found that the longer the active medium, the higher the buffer gas pressure and pump radiation intensity, and the smaller the width of the pump radiation spectrum, the greater the efficiency of conversion of pump radiation into laser radiation. In a sufficiently long active medium (the length of the medium is 50 times its width), the conversion efficiency can reach 44% at a buffer gas pressure of 5 atm, a pump diode radiation intensity of 3 kW cm⁻², and a half-width of its spectrum of 1 cm⁻¹. A two-level gas laser with transverse diode pumping is capable of generating continuous optical radiation with a very high (up to 100 kW) power. The frequency of laser radiation can be tuned by several tens of cm⁻¹.

Keywords: two-level system, inversionless radiation amplification, diode pumping, collisions, Einstein coefficients, spectral line wing.

1. Introduction

Diode-pumped alkali metal vapour lasers have been intensively studied in the last decade (see, for example, [1-6] and references therein). To date, these lasers have demonstrated their high efficiency, which made it possible to develop highpower laser systems operating according to the standard three-level V-scheme with close upper levels between which active collisional mixing occurs.

As it turned out, it is possible to reduce the number of levels to two and still obtain lasing in the 'red' wing of the spectral line under resonant diode pumping [7]. Laser generation is possible due to collisions, when the probabilities of absorption and stimulated emission in the wing of the spectral line cease to be equal to each other.

Previously, it was shown in Refs [8-19] that in the wing the absorption lines of active gas particles in the presence of

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Received 8 December 2021 *Kvantovaya Elektronika* **52** (5) 426–436 (2022) Translated by I.A. Ulitkin frequent collisions with buffer particles, the probabilities of absorption and stimulated emission are not equal to each other. It turned out that the spectral densities of the Einstein coefficients for absorption $[b_{12}(\Omega)]$ and stimulated emission $[b_{21}(\Omega)]$ are related by the expression [14, 15]

$$b_{12}(\Omega) = b_{21}(\Omega) \exp\left(\frac{\hbar\Omega}{k_{\rm B}T}\right),\tag{1}$$

where $\Omega = \omega - \omega_{21}$ is the detuning of the radiation frequency ω from the frequency ω_{21} of the 2–1 transition; \hbar is Planck's constant; $k_{\rm B}$ is the Boltzmann constant; and *T* is the temperature. Relation (1) remains valid for any sign of Ω .

Markov et al. [13, 15-17] experimentally demonstrated the formation of population inversion in a two-level system upon absorption of intense radiation in the blue wing of the spectral line and, as a consequence, the occurrence of lasing at the resonant frequency. This effect is explained by the action of relation (1). However, in works [13, 15-17] lasing was obtained by employing a pulsed pump laser with a very high peak power. It is currently impossible to use such an experimental setup to convert incoherent radiation into coherent (laser) radiation, since existing sources (including laser diodes) do not have sufficient intensity for this. If, nevertheless, we set ourselves such a task, then another formulation of the experiment turns out to be fruitful.

One of the consequences of relation (1) is the amplification of radiation by two-level systems without population inversion. If high-power pump radiation, tuned in frequency to resonance with the atomic transition, equalises the populations of the excited and ground levels, then due to an excess of the probability of stimulated emission over the probability of absorption in the 'red' wing of the spectral line (at $\Omega < 0$), the conditions for radiation amplification are realised [2, 7, 14, 20]. Modern systems of diode pumping, when tuned to resonance with the transition, are quite capable of equalising the populations of the levels. Since the gain for the generated radiation during diode pumping turns out to be rather small [7], a long path along the amplifying medium is needed in order to achieve a noticeable gain in the active medium per pass. This can be implemented using transverse diode pumping of the active medium. In this paper, we theoretically consider the operation of a two-level gas laser with transverse diode pumping.

2. Equations describing the operation of a laser

Consider a gas of two-level absorbing particles (with a ground level 1 and an excited level 2) in a mixture with a buffer gas. We neglect collisions between absorbing particles, assuming that the concentration of the buffer gas, $N_{\rm b}$, is much higher than the concentration of the absorbing gas, N. Let the twolevel particles be affected by high-power (capable of levelling the populations of levels 1 and 2) pump diode radiation, tuned in frequency to resonance with the atomic transition. We assume that the radiation of the pump diodes has a spectrum of arbitrary width, and the radiation generated in the 'red' wing of the spectral line is monochromatic.

Let us consider the operation of a two-level gas laser with transverse diode pumping, the schematic of which is shown in Fig. 1. To simplify the analysis, we assumed that a cell with active particles and a buffer gas has the shape of a rectangular parallelepiped with edge lengths z_0 (length), y_0 (width), and x_0 (height). The pump laser diodes are located on one side of the cell. Their radiation enters the cell through the xz plane and propagates in the direction of the y axis. For a more complete use of the pump radiation energy, a flat mirror is installed on the other side of the cell, which returns the pump radiation that has passed through it back to the cell (mirror reflection coefficient R_p). The resonator consists of two mirrors with reflection coefficients R_0 and R_1 . The radiation energy loss in the resonator, which consists of diffraction losses, losses at the cell windows, and losses associated with the geometrical imperfection of the resonator, will be taken into account by introducing the effective transmittance $T_{\rm r}$, the value of which characterises the relative loss of radiation energy in the resonator per single pass. We assume that the losses T_r are localised in front of the rear mirror having the reflection coefficient R_1 . Laser radiation leaves the cell in the xy plane through a mirror with a reflection coefficient R_0 and propagates in the direction of the z axis. For simplicity, we assume that the distribution of the pump radiation intensity is uniform in the xz plane (at the cell entrance). In this case, as a consequence, the laser radiation intensity distribution is also uniform over the cell height (along the x axis).



Figure 1. Schematic of a two-level gas laser with transverse diode pumping.

Under stationary conditions, the absorption of pump radiation and amplification of laser radiation in the red wing of the spectral line (at $\Omega_{las} < 0$) are described by the equations:

$$\frac{\partial I_{\omega p}^{\pm}(y, z, \omega)}{\partial y} = \mp \left[N_{1}(y, z) - N_{2}(y, z) \right] \sigma_{p}(\omega) I_{\omega p}^{\pm}(y, z, \omega),$$

$$\frac{\partial I_{\text{las}}^{\pm}(y, z)}{\partial z} = \mp \left[\xi_{\text{las}} N_{1}(y, z) - N_{2}(y, z) \right] \sigma_{\text{las}}(\omega_{\text{las}}) I_{\text{las}}^{\pm}(y, z),$$
(2)

where we introduce the notation

$$\xi_{\rm las} = \exp\left(\frac{\hbar\Omega_{\rm las}}{k_{\rm B}T}\right), \quad \Omega_{\rm las} = \omega_{\rm las} - \omega_{21}. \tag{3}$$

Here ω_{21} and ω_{las} are the frequencies of the 2–1 transition and generated laser radiation, respectively; $I_{\omega p}^+(y, z, \omega)$ and $I_{\omega p}^-(y, z, \omega)$ are the spectral densities of the pump radiation intensity at the frequency ω , propagating along the y axis and in the opposite direction (after reflection from the mirror), respectively; $I_{\text{las}}^+(y, z)$ and $I_{\text{las}}^-(y, z)$ are the intensities of laser radiation propagating along the z axis and in the opposite direction, respectively; $N_1(y, z)$ and $N_2(y, z)$ are the populations of levels 1 and 2; $\sigma_p(\omega)$ is the absorption cross section of pump radiation; and $\sigma_{\text{las}}(\omega_{\text{las}})$ is the stimulated emission cross section with the emission of a photon with frequency ω_{las} . The cross sections are found from the formulae:

$$\sigma_{\rm p}(\omega) = \frac{\lambda_{21}^2 A_{21}}{4\pi} \frac{\Gamma}{\Gamma^2 + (\omega - \omega_{21})^2},$$

$$\sigma_{\rm las}(\omega_{\rm las}) = \frac{\lambda_{21}^2 A_{21}}{4\pi} \frac{\Gamma_{\rm oc}(\Omega_{\rm las})}{\Gamma^2 + (\omega_{\rm las} - \omega_{21})^2},$$
(4)

where A_{21} is the rate of spontaneous emission (the first Einstein coefficient) for the 2–1 transition; λ_{21} is the wavelength; $\Gamma = A_{21}/2 + \gamma$ is the homogeneous half-width of the transition line 2–1; and γ is the collisional half-width of the line of the given transition. The quantity $\Gamma_{\rm oc}(\Omega_{\rm las})$, which depends on the detuning of the laser radiation frequency $\Omega_{\rm las}$ from the frequency of the 2-1 transition, characterises the frequency of elastic collisions that knock the phase of the atomic oscillator [21]. The quantity $\Gamma_{\rm oc}(\Omega_{\rm las})$ is included in the modified Lorentz formula, which describes the entire contour of the spectral line, including the distant wings [21]. At a small frequency detuning of the laser radiation ($|\Omega_{las}| \leq \Gamma$), the value of $arGamma_{
m oc}(arOmega_{
m las})$ is equal to the homogeneous half-width of the absorption line Γ , and at a large frequency detuning $(|\Omega_{\rm las}| \gg \Gamma$, absorption line wing), the value of $\Gamma_{\rm oc}(\Omega_{\rm las})$ can be both much larger and much smaller than Γ [21]. Equations (2) are supplemented with boundary conditions that determine the change in the radiation intensities on the surfaces of the mirrors:

$$I^{+}_{\omega p}(0, z, \omega) = I_{0\omega p}(\omega),$$

$$I^{-}_{\omega p}(y_{0}, z, \omega) = R_{p}I^{+}_{\omega p}(y_{0}, z, \omega),$$

$$I^{+}_{las}(y, 0) = R_{1}T_{r}^{2}I^{-}_{las}(y, 0),$$

$$I^{-}_{las}(y, z_{0}) = R_{0}I^{+}_{las}(y, z_{0}).$$
(5)

The populations of levels 1 and 2 in equations (2) are found from the balance equations. These equations in the stationary case are written as

$$\frac{\mathrm{d}N_2}{\mathrm{d}t} = 0 = -A_{21}N_2(y,z) + w_{\rm p}(y,z)[N_1(y,z) - N_2(y,z)] + w_{\rm las}(y,z)[\xi_{\rm las}N_1(y,z) - N_2(y,z)],$$
(6)

 $N_1(y,z) + N_2(y,z) = N.$

Here *N* is the total concentration of active atoms; and $w_p(y,z)$ and $w_{las}(y,z)$ are the probabilities of stimulated transitions under the action of pump radiation and generated laser radiation, respectively. We assume that the pump radiation has a spectrum of arbitrary width, and the generated laser radiation is monochrome. Then

$$w_{\rm p}(y,z) = \int_0^\infty \frac{\sigma_{\rm p}(\omega)}{\hbar\omega_{\rm p}} I_{\omega \rm p}(y,z,\omega) \,\mathrm{d}\omega,$$

$$w_{\rm las}(y,z) = \frac{\sigma_{\rm las}(\omega_{\rm las})}{\hbar\omega_{\rm las}} I_{\rm las}(y,z),$$
(7)

where $\omega_{\rm p}$ is the centre frequency of the pump radiation spectrum; and

$$I_{\omega p}(y, z, \omega) = I^{+}_{\omega p}(y, z, \omega) + I^{-}_{\omega p}(y, z, \omega),$$

$$I_{las}(y, z) = I^{+}_{las}(y, z) + I^{-}_{las}(y, z)$$
(8)

are the total spectral density of the pump radiation intensity inside the cell and the total intensity of the laser radiation inside the cell, respectively. From the system of algebraic equations (6) we find the level populations:

$$N_{1}(y,z) = \frac{N}{2} \frac{2 + \varkappa_{p} + \frac{2\varkappa_{las}}{1 + \xi_{las}}}{1 + \varkappa_{p} + \varkappa_{las}},$$

$$N_{2}(y,z) = \frac{N}{2} \frac{\varkappa_{p} + \frac{2\varkappa_{las}\xi_{las}}{1 + \xi_{las}}}{1 + \varkappa_{p} + \varkappa_{las}}.$$
(9)

We also present expressions for combinations of populations that characterise lasing and pump absorption:

$$N_{2}(y,z) - \xi_{\rm las} N_{1}(y,z) = N \frac{\varkappa_{\rm p} \frac{1 - \xi_{\rm las}}{2} - \xi_{\rm las}}{1 + \varkappa_{\rm p} + \varkappa_{\rm las}},$$

$$N_{1}(y,z) - N_{2}(y,z) = N \frac{\varkappa_{\rm las} \frac{1 - \xi_{\rm las}}{1 + \xi_{\rm las}} + 1}{1 + \varkappa_{\rm p} + \varkappa_{\rm las}}.$$
(10)

The quantities $\varkappa_{p} \equiv \varkappa_{p}(y, z)$ and $\varkappa_{las} \equiv \varkappa_{las}(y, z)$ defined as

$$\varkappa_{\rm p} = \frac{2w_{\rm p}(y,z)}{A_{21}},$$
(11)
$$\varkappa_{\rm las} = \frac{(1+\xi_{\rm las})w_{\rm las}(y,z)}{A_{21}},$$

are the saturation parameters, since each of them characterises the degree of population equalisation at the 2-1 transition in the absence of the second field. Taking into account relations (10), differential equations (2) describing the laser operation are written in the form:

$$\frac{\partial I_{\omega p}^{\pm}(y,z,\omega)}{\partial y} = \mp \frac{\left[1 + \varkappa_{\rm las} \frac{1 - \xi_{\rm las}}{1 + \xi_{\rm las}}\right] N \sigma_{\rm p}(\omega) I_{\omega p}^{\pm}(y,z,\omega)}{1 + \varkappa_{\rm p} + \varkappa_{\rm las}},$$

$$\frac{\partial I_{\rm las}^{\pm}(y,z)}{\partial z} = \pm \frac{\left[\varkappa_{\rm p} \frac{1 - \xi_{\rm las}}{2} - \xi_{\rm las}\right] N \sigma_{\rm las}(\omega_{\rm las}) I_{\rm las}^{\pm}(y,z)}{1 + \varkappa_{\rm p} + \varkappa_{\rm las}}.$$
(12)

3. Relationship between integral characteristics of the radiations

The system of differential equations (12) describing the laser operation can only be solved numerically. Nevertheless, it is possible, without solving it, to obtain a practically important relationship between the integral characteristics of the radiations.

For the power of laser radiation generated inside the cell, from the first equation (6), taking into account the first formula (10), we obtain the expression (integration is performed over the cell volume):

$$P_{\rm las}^{\rm cell} = \hbar\omega_{\rm las} \int w_{\rm las} (N_2 - \xi_{\rm las} N_1) \,\mathrm{d}V$$
$$= \frac{N\hbar\omega_{\rm las} A_{21}}{1 + \xi_{\rm las}} x_0 \int_0^{y_0} \mathrm{d}y \int_0^{z_0} \mathrm{d}z \frac{\varkappa_{\rm las} \left[\varkappa_{\rm p} \frac{1 - \xi_{\rm las}}{2} - \xi_{\rm las}\right]}{1 + \varkappa_{\rm p} + \varkappa_{\rm las}}. (13)$$

We assume hereafter that in the considered geometry of a laser with transverse pumping, laser radiation is generated in the entire cell volume $V = x_0y_0z_0$. For further calculations, we need one more expression for the power of laser radiation generated inside the cell:

$$P_{\text{las}}^{\text{cell}} = x_0 \int_0^{y_0} [I_{\text{las}}^+(y, z_0) - I_{\text{las}}^-(y, z_0) + I_{\text{las}}^-(y, 0) - I_{\text{las}}^+(y, 0)] dy.$$
(14)

The intensity $I_{\text{las}}^{\text{out}}(y)$ of laser radiation emerging from the resonator through a mirror with a reflection coefficient R_0 is written in the form

$$I_{\rm las}^{\rm out}(y) = (1 - R_0) I_{\rm las}^+(y, z_0).$$
(15)

The power $P_{\text{las}}^{\text{out}}$ of the laser radiation emerging from the resonator through an output mirror is determined as the integral of the intensity over the beam cross section:

$$P_{\text{las}}^{\text{out}} = x_0 \int_0^{y_0} I_{\text{las}}^{\text{out}}(y) dy.$$
(16)

Let us find the relation between the powers $P_{\text{las}}^{\text{out}}$ (16) and $P_{\text{las}}^{\text{cell}}$ (13). From the last two equations in (12) we obtain the expression

$$\frac{1}{I_{\rm las}^+(y,z)}\frac{\partial I_{\rm las}^+(y,z)}{\partial z} = -\frac{1}{I_{\rm las}^-(y,z)}\frac{\partial I_{\rm las}^-(y,z)}{\partial z};$$
(17)

whence it follows that the product $I_{las}^+(y,z)I_{las}^-(y,z)$ is independent of the *z* coordinate. In particular, the relation

$$I_{\rm las}^+(y,z_0)I_{\rm las}^-(y,z_0) = I_{\rm las}^+(y,0)I_{\rm las}^-(y,0)$$
(18)

is valid. For the integrand in (14), taking into account (15), (18) and boundary conditions (5), we have

$$I_{\rm las}^{+}(y, z_0) - I_{\rm las}^{-}(y, z_0) + I_{\rm las}^{-}(y, 0) - I_{\rm las}^{+}(y, 0) = \frac{I_{\rm las}^{\rm out}(y)}{R},$$
(19)

where

$$R = \frac{T_{\rm r}(1-R_0)\sqrt{R_1}}{T_{\rm r}(1-R_0)\sqrt{R_1} + (1-R_1T_{\rm r}^2)\sqrt{R_0}}.$$
(20)

Expressions (13), (14), (16), and (19) yield the relationship between the powers $P_{\text{las}}^{\text{out}}$ and $P_{\text{las}}^{\text{cell}}$:

$$P_{\rm las}^{\rm out} = RP_{\rm las}^{\rm cell} = \frac{RN\hbar\omega_{\rm las}A_{21}}{1+\xi_{\rm las}} \times x_0 \int_0^{y_0} dy \int_0^{z_0} dz \frac{\varkappa_{\rm las} \left[\varkappa_{\rm p} \frac{1-\xi_{\rm las}}{2} - \xi_{\rm las}\right]}{1+\varkappa_{\rm p} + \varkappa_{\rm las}}.$$
 (21)

This formula is the usual relationship for the energy balance: the power of the output laser radiation is equal to the energy generated in the active medium per unit time, minus the losses of this energy inside the resonator.

For the absorbed power of the pump radiation, from the first equation (6), taking into account the second formula (10), we obtain

$$P_{\rm abs} = \hbar \omega_{\rm p} \int w_{\rm p} [N_1 - N_2] \,\mathrm{d}V$$
$$= N \hbar \omega_{\rm p} \frac{A_{21}}{2} x_0 \int_0^{y_0} \mathrm{d}y \int_0^{z_0} \mathrm{d}z \frac{\varkappa_{\rm p} \left[\varkappa_{\rm las} \frac{1 - \xi_{\rm las}}{1 + \xi_{\rm las}} + 1\right]}{1 + \varkappa_{\rm p} + \varkappa_{\rm las}}.$$
 (22)

Next, we take into account the energy losses due to spontaneous emission in the cell volume. These losses are described by the fairly obvious expression:

$$P_{\text{loss}} = \hbar \omega_{\text{p}} \int N_2 A_{21} dV$$

= $N \hbar \omega_{\text{p}} \frac{A_{21}}{2} x_0 \int_0^{y_0} dy \int_0^{z_0} dz \frac{\varkappa_{\text{p}} + \frac{2\varkappa_{\text{las}} \xi_{\text{las}}}{1 + \xi_{\text{las}}}}{1 + \varkappa_{\text{p}} + \varkappa_{\text{las}}}.$ (23)

From (21), (22), and (23) we obtain the expression

$$P_{\rm las}^{\rm out} = R \frac{\omega_{\rm las}}{\omega_{\rm p}} [P_{\rm abs} - P_{\rm loss}], \tag{24}$$

which relates the power of the laser radiation emerging from the resonator, $P_{\text{las}}^{\text{out}}$, with the absorbed power of the pump

emission. Energy losses also arise due to incomplete absorption of pump radiation:

$$P_{\rm unabs} = P_{\rm 0p} - P_{\rm abs},\tag{25}$$

where P_{0p} is the pump radiation power at the cell input; and P_{unabs} is the power of the unabsorbed part of the pump radiation. Taking into account (25), relation (24) for the output power of laser radiation can also be expressed as

$$\frac{P_{\rm las}^{\rm out}}{P_{\rm 0p}} = R \frac{\omega_{\rm las}}{\omega_{\rm p}} \left(1 - \frac{P_{\rm unabs} + P_{\rm loss}}{P_{\rm 0p}} \right).$$
(26)

The ratio of the laser radiation power to the pump radiation power $P_{\rm las}^{\rm out}/P_{\rm 0p}$ characterises the conversion efficiency of pump radiation into laser radiation, and the ratio of radiation frequencies $\omega_{\rm las}/\omega_{\rm p}$ characterises the quantum efficiency of conversion of pump radiation into laser radiation. For a twolevel gas laser, the quantum efficiency is close to unity. The efficiency of conversion of pump radiation into laser radiation $P_{\rm las}^{\rm out}/P_{\rm 0p}$ is the higher, the smaller the relative energy losses ($P_{\rm unabs} + P_{\rm loss}$)/ $P_{\rm 0p}$ and the closer to unity the coefficient R, which characterises the losses of the generated radiation on the mirrors, cell windows and in the amplifying medium.

4. Analytical solution of the problem in the approximation of level populations independent of the coordinate z

For a not too small reflection coefficient of the output mirror R_0 , the populations of the levels of the active medium atoms, as will be shown below, are practically independent of the *z* coordinate along the resonator axis:

$$N_1(y,z) \equiv N_1(y),$$

$$N_2(y,z) \equiv N_2(y).$$
(27)

In this approximation, the system of differential equations (12) describing the laser operation is greatly simplified and admits an analytical solution, which makes it possible to exhaustively determine any energy characteristics of the laser. In approximation (27), the right-hand sides in relations (10) depend only on the y coordinate. Equations (12) in approximation (27) take the form:

$$\frac{\partial I_{\omega p}^{\pm}(y,\omega)}{\partial y} = \mp A_{p1}(y) N \sigma_{p}(\omega) I_{\omega p}^{\pm}(y,\omega),$$

$$\frac{\partial I_{\text{las}}^{\pm}(y,z)}{\partial z} = \pm g_{\text{las}}(y) I_{\text{las}}^{\pm}(y,z),$$
(28)

where we introduce the notations

$$A_{\rm p1}(y) = \frac{1 + \varkappa_{\rm las} \frac{1 - \xi_{\rm las}}{1 + \xi_{\rm las}}}{1 + \varkappa_{\rm p} + \varkappa_{\rm las}},\tag{29}$$

$$g_{\rm las}(y) = \frac{\varkappa_{\rm p} \frac{1 - \xi_{\rm las}}{2} - \xi_{\rm las}}{1 + \varkappa_{\rm p} + \varkappa_{\rm las}} N \sigma_{\rm las}(\omega_{\rm las}).$$
(30)

The solution of the second equation in (28) has the form

$$I_{\rm las}^{\pm}(y,z) = c^{\pm}(y) \exp[\pm g_{\rm las}(y)z],$$
(31)

where $c^{\pm}(y)$ is the integration constant depending on the y coordinate. From (31), taking into account the boundary conditions (5), we obtain the relation

$$g_{\rm las}(y) = g_0,\tag{32}$$

where

$$g_0 = \frac{1}{2z_0} \ln \frac{1}{R_0 R_1 T_r^2}$$
(33)

is threshold value of the gain. The gain $g_{\text{las}}(y)$ of the medium in the resonator under stationary generation conditions does not depend on y and is equal to the threshold value of g_0 . From relation (32), taking into account (30) for $g_{\text{las}}(y)$, we obtain the relationship between the saturation parameters \varkappa_{las} and \varkappa_{p} :

$$\kappa_{\rm las} = \kappa_{\rm p} \left[\frac{(1 - \xi_{\rm las}) N \sigma_{\rm las}(\omega_{\rm las})}{2g_0} - 1 \right] - \frac{\xi_{\rm las} N \sigma_{\rm las}(\omega_{\rm las})}{g_0} - 1.$$
(34)

The function $A_{pl}(y)$ (29) with \varkappa_{las} (34) substituted into it turns out to be independent of *y*:

$$A_{\rm pl}(y) = A_{\rm p},\tag{35}$$

where

$$A_{\rm p} = \frac{1 - \xi_{\rm las}}{1 + \xi_{\rm las}} - \frac{2g_0}{(1 + \xi_{\rm las})N\sigma_{\rm las}(\omega_{\rm las})}.$$
 (36)

Taking into account (35), the first equation in (28) takes the form

$$\frac{\partial I_{\omega p}^{\pm}(y,\omega)}{\partial y} = \mp A_{p} N \sigma_{p}(\omega) I_{\omega p}^{\pm}(y,\omega).$$
(37)

Then, taking into account the boundary conditions (5), we find the solution to equation (37):

$$I_{\omega p}^{+}(y,\omega) = I_{0\omega p}(\omega) \exp[-A_{p}\sigma_{p}(\omega)Ny],$$

$$I_{\omega p}^{-}(y,\omega) = R_{p}I_{0\omega p}(\omega) \exp[-A_{p}\sigma_{p}(\omega)N(2y_{0}-y)].$$
(38)

According to (38), the spectral density of the pump radiation intensity decreases exponentially as it passes through the cell medium. This circumstance is due to the fact that, under the conditions in question, the difference in the populations of the $N_1 - N_2$ levels, which determines the pump absorption, does not depend on the intensities of the pump and laser radiation: $N_1 - N_2 = NA_p$.

The saturation parameter \varkappa_p depends on the *y* coordinate, is independent of the *z* coordinate, and, in accordance with (7), (8), and (11), is determined by the expression

$$\varkappa_{\rm p}(y) = \frac{2}{A_{21}\hbar\omega_{\rm p}} \int_0^\infty \sigma_{\rm p}(\omega) [I^+_{\omega \rm p}(y,\omega) + I^-_{\omega \rm p}(y,\omega)] \,\mathrm{d}\omega. \tag{39}$$

Let us use formula (21) to find the power $P_{\text{las}}^{\text{out}}$ of the laser radiation emerging from the resonator through the output mirror with the reflection coefficient R_0 . For the integrand in (21), taking into account (34), we have

$$\frac{\varkappa_{\text{las}} \left[\varkappa_{\text{p}} \frac{1 - \xi_{\text{las}}}{2} - \xi_{\text{las}} \right]}{1 + \varkappa_{\text{p}} + \varkappa_{\text{las}}} = \left[\frac{1 - \xi_{\text{las}}}{2} - \frac{g_0}{N \sigma_{\text{las}}(\omega_{\text{las}})} \right] \varkappa_{\text{p}}(y)$$
$$-\xi_{\text{las}} - \frac{g_0}{N \sigma_{\text{las}}(\omega_{\text{las}})}. \tag{40}$$

From (21), taking into account (40), for the power $P_{\text{las}}^{\text{out}}$ we obtain the expression:

$$P_{\text{las}}^{\text{out}} = RNx_0 z_0 \frac{\hbar\omega_{\text{las}}A_{21}}{1 + \xi_{\text{las}}} \left\{ \left[\frac{1 - \xi_{\text{las}}}{2} - \frac{g_0}{N\sigma_{\text{las}}(\omega_{\text{las}})} \right] \right\}$$
$$\times \int_0^{y_0} \varkappa_p(y) \, \mathrm{d}y - y_0 \left[\xi_{\text{las}} + \frac{g_0}{N\sigma_{\text{las}}(\omega_{\text{las}})} \right] \right\}.$$
(41)

Comparing formulae (16) and (41), we find the intensity of the output laser radiation $I_{\text{las}}^{\text{out}}(y)$:

$$I_{\text{las}}^{\text{out}}(y) = RNz_0 \frac{\hbar\omega_{\text{las}}A_{21}}{1 + \xi_{\text{las}}}$$
$$\times \left\{ \left[\frac{1 - \xi_{\text{las}}}{2} - \frac{g_0}{N\sigma_{\text{las}}(\omega_{\text{las}})} \right] \varkappa_p(y) - \xi_{\text{las}} - \frac{g_0}{N\sigma_{\text{las}}(\omega_{\text{las}})} \right\}.$$
(42)

For the absorbed pump radiation power, from formula (22), taking into account (34), we obtain the expression:

$$P_{\rm abs} = N x_0 z_0 \frac{\hbar \omega_{\rm p} A_{21}}{1 + \xi_{\rm las}}$$
$$\times \left[\frac{1 - \xi_{\rm las}}{2} - \frac{g_0}{N \sigma_{\rm las}(\omega_{\rm las})} \right] \int_0^{y_0} \varkappa_{\rm p}(y) \, \mathrm{d}y. \tag{43}$$

For the energy losses due to spontaneous emission in the cell volume, from (23), taking into account (34), we have

$$P_{\rm loss} = NV \frac{\hbar\omega_{\rm p} A_{21}}{1 + \xi_{\rm las}} \bigg[\xi_{\rm las} + \frac{g_0}{N\sigma_{\rm las}(\omega_{\rm las})} \bigg].$$
(44)

In conclusion, let us consider the condition for the applicability of approximation (27) that the populations of the levels of atoms of the active medium are independent of the zcoordinate. To fulfil condition (27), it is necessary that the total intensity of laser radiation inside the cell, $I_{\text{las}}(y,z)$ (8) along the *z* axis change insignificantly. For the intensity of laser radiation inside the cell, from (31) and (32), taking into account the boundary conditions (5), we obtain

$$I_{\rm las}(y,z) = c^{-}(y)[R_1 T_{\rm r}^2 \exp(g_0 z) + \exp(-g_0 z)].$$
(45)

Taking this expression as a basis, we can estimate the inhomogeneity of the radiation field in the cell as the ratio of the minimum (I_{las}^{\min}) and maximum (I_{las}^{\max}) intensity values on the *z* axis:

$$\frac{I_{\rm las}^{\rm min}}{I_{\rm las}^{\rm max}} = \frac{2\sqrt{R_0}}{1+R_0}.$$
(46)

Formula (46) is valid for $R_0 < R_1 T_r^2$. According to (46), at $R_0 > 0.6$, the intensity $I_{\text{las}}(y, z)$ along the *z* axis varies insignificantly ($I_{\text{las}}^{\min}/I_{\text{las}}^{\max} = 0.97$ at $R_0 = 0.6$), which makes it possible to assume, in a good approximation, that the populations of the levels of the active medium atoms are independent of the *z* coordinate.

5. Analysis of the generation characteristics of the laser

To further specify the calculations using the above formulae, it is necessary to set the spectral density of the radiation intensity $I_{0\omega p}(\omega)$ of the pump diodes at the cell input. We will assume that the pump radiation spectrum at the cell input has a Gaussian shape:

$$I_{0\omega p}(\omega) = \frac{I_{0p}}{\sqrt{\pi} \Delta \omega} \exp\left[-\left(\frac{\omega - \omega_{p}}{\Delta \omega}\right)^{2}\right],$$

$$I_{0p} = \int_{0}^{\infty} I_{0\omega p}(\omega) d\omega,$$
(47)

where I_{0p} is the pump radiation intensity at the cell input; and $\Delta \omega$ is the half-width (at a height 1/e) of the pump radiation spectrum. From (38), taking into account (47), for the total spectral density of the pump radiation intensity inside the cell, $I_{\omega p}(y, \omega)$, we obtain

$$I_{\omega p}(y,\omega) = I^{+}_{\omega p}(y,\omega) + I^{-}_{\omega p}(y,\omega) = \frac{I_{0p}}{\sqrt{\pi}\Delta\omega}$$
$$\times \{\exp[-g(\omega, y)] + R_{p}\exp[-g(\omega, 2y_{0} - y)]\}, \qquad (48)$$

$$g(\omega, y) = \left(\frac{\omega - \omega_{\rm p}}{\Delta \omega}\right)^2 + A_{\rm p}\sigma_{\rm p}(\omega)Ny.$$

From here we find the total intensity of the pump radiation inside the cell:

$$I_{p}(y) = I_{p}^{+}(y) + I_{p}^{-}(y) = I_{0p}[f_{1}(y) + R_{p}f_{1}(2y_{0} - y)],$$

$$f_{1}(y) = \frac{1}{\sqrt{\pi}\Delta\omega} \int_{0}^{\infty} \exp[-g(\omega, y)] d\omega.$$
(49)

The saturation parameter $\varkappa_p(y)$ under the conditions in question is defined as

$$\kappa_{\rm p}(y) = \frac{2\sigma_{\rm p}(\omega_{21})}{A_{21}\hbar\omega_{\rm p}} I_{0\rm p}[f_2(y) + R_{\rm p}f_2(2y_0 - y)],$$

$$f_2(y) = \frac{1}{\sqrt{\pi}\Delta\omega} \int_0^\infty \frac{\exp[-g(\omega, y)]}{1 + [(\omega - \omega_{21})/\Gamma]^2} d\omega.$$
(50)

For the absorbed power of pump radiation, from expression (43), taking into account (49) and (50), we obtain the relation

$$P_{\rm abs} = P_{\rm 0p} \{ 1 - f_1(y_0) + R_{\rm p} [f_1(y_0) - f_1(2y_0)] \},$$
(51)

where $P_{0p} = x_0 z_0 I_{0p}$ is the pump radiation power at the cell input. From formula (24), using (51) and (44), we obtain the expression for the power of laser radiation emerging from the resonator:

$$P_{\rm las}^{\rm out} = R \frac{\omega_{\rm las}}{\omega_{\rm p}} P_{\rm 0p} \{ 1 - f_{\rm 1}(y_0) + R_{\rm p} [f_{\rm 1}(y_0) - f_{\rm 1}(2y_0)] \}$$
$$- RNV \frac{\hbar \omega_{\rm las} A_{21}}{1 + \xi_{\rm las}} \Big[\xi_{\rm las} + \frac{g_0}{N\sigma_{\rm las}(\omega_{\rm las})} \Big].$$
(52)

For the ratio of the power of laser radiation emerging from the resonator to the power of pump radiation, which characterises the efficiency of converting pump radiation into laser radiation, we have

$$\frac{P_{\rm las}^{\rm out}}{P_{\rm 0p}} = R \frac{\omega_{\rm las}}{\omega_{\rm p}} \{1 - f_{\rm l}(y_0) + R_{\rm p} [f_{\rm l}(y_0) - f_{\rm l}(2y_0)]\} - \frac{RNy_0 \hbar \omega_{\rm las} A_{21}}{I_{\rm 0p} (1 + \xi_{\rm las})} \bigg[\xi_{\rm las} + \frac{g_0}{N\sigma_{\rm las}(\omega_{\rm las})} \bigg].$$
(53)

Let us also present an expression for the power of the unabsorbed part of the pump radiation:

$$P_{\text{unabs}} = P_{0p} - P_{\text{abs}} = P_{0p} \{ f_1(y_0) - R_p [f_1(y_0) - f_1(2y_0)] \}.$$
(54)

Let us use the above formulae to calculate the energy characteristics of a two-level laser. Let caesium atoms be the active medium in the laser cell, and helium be used as a buffer gas. Radiation is generated in the red wing of the D_1 line of caesium atoms ($6^2S_{1/2} - 6^2P_{1/2}$ transition).

To calculate the characteristics of radiation generation in the red wing of the D₁ line of caesium atoms, one can use the two-level model of absorbing particles due to the weak collisional coupling between the fine components $6^2P_{1/2}$ and $6^2P_{3/2}$ of the excited state of Cs atoms in He. Indeed, the cross section $\sigma_{\rm FS}$ of $6^2P_{1/2} \rightarrow 6^2P_{3/2}$ collisional transitions between fine components of the excited state of Cs atoms in He is small: $\sigma_{\rm FS} = 0.57 \times 10^{-20}$ cm² [22]. At a helium pressure $p_{\rm He} = 10$ atm and a temperature T = 430 K, the frequency of $6^2P_{1/2} \rightarrow 6^2P_{3/2}$ collisional transitions between fine components is $v_{\rm FS} =$ $1.5 \times 10^5 \text{ s}^{-1}$, which is much smaller than the rate of spontaneous decay of the excited $6^2 P_{1/2}$: $v_{FS}/A_{21} = 5.2 \times 10^{-3}$. On this basis, the $6^2 P$ excited state of Cs atoms can be modelled by a single level. The ground level $6^2 S_{1/2}$ of Cs atoms is split into two hyperfine components with a frequency spacing $\Delta \omega_{HFS} =$ $5.78 \times 10^{10} \text{ c}^{-1} [\Delta \omega_{HFS}/(2\pi c) = 0.31 \text{ cm}^{-1}]$ [23]. At a sufficiently high pressure of the buffer gas (several atmospheres or higher), the collisional width of the absorption line is large compared to the hyperfine splitting of this state: $2\Gamma \gg$ $\Delta \omega_{HFS}$. Therefore, the ground state can also be modelled with a single level.

Let us set the initial data necessary for calculating the operation of the laser. According to the data of the NIST website [24], the rate of spontaneous decay of the excited level $6^2P_{1/2}$ of the caesium atom is $A_{21} = 2.86 \times 10^7 \text{ s}^{-1}$, the wavelength of the D₁ line is $\lambda_{21} = 894.4 \text{ nm}$. The collisional broadening coefficient for the D₁ line of caesium atoms in a helium buffer gas is 11.58 MHz Torr⁻¹ at a temperature T = 430 K [25].

Then, we assume in the calculation that the centre frequency of the pump radiation spectrum coincides with the frequency of the 2–1 transition: $\omega_p = \omega_{21}$. It is advisable to design the cell in such a way that alkali metal vapour enters the cell through the lateral processes. In this case, the concentration N of active particles inside the cell is determined by the temperature of the lateral processes containing the alkali metal and is not related to the temperature T of the gas mixture inside the cell.

To calculate the characteristics of radiation generation in the red wing of the D_1 line of caesium atoms, it is necessary to know the rate of phase relaxation $\Gamma_{\rm oc}(\Omega_{\rm las})$ during their collisions with buffer gas molecules. In [26], Fig. 13 shows the calculated profile of the D_1 line of Cs atoms in a He buffer gas at a pressure of 13.6 atm and a temperature T = 1000 K in comparison with its Lorentzian profile. It follows from this figure that in the red wing of the D_1 line, $\Gamma_{\rm oc}(\Omega_{\rm las}) \ge \Gamma$ at frequency detunings $\Omega_{\rm las}/(2\pi c)$ from 0 to -1100 cm⁻¹. Therefore, we will assume in calculations below that the rate of phase relaxation during collisions is equal to the homogeneous half-width of the absorption line: $\Gamma_{\rm oc}(\Omega_{\rm las}) = \Gamma$.

Figure 2 shows the results of calculations [using formula (53)] of the conversion efficiency of pump radiation into laser radiation $P_{\text{las}}^{\text{out}}/P_{0p}$ as a function of the frequency detuning Ω_{las} for different ratios of the cell length to its width z_0/y_0 . We assumed that at the cell input, the pump radiation intensity is $I_{0p} = 3 \text{ kW cm}^{-2}$, its spectrum half-width is $\Delta \omega / (2\pi c) = 1 \text{ cm}^{-1}$, and the buffer gas pressure is $p_{\text{He}} = 5$ atm. When calculating each curve in Fig. 2, we set such values of the parameter Ny_0 (the number of active atoms in a cell in a gas column of height y_0 with a unit cross section) and the reflection coefficient R_0 of the output mirror at which the maximum of the ratio $P_{\rm las}^{\rm out}/P_{\rm 0p}$ as a function of $\Omega_{\rm las}$ has the largest value (below such values of the parameter Ny_0 and the reflection coefficient R_0 will be called optimal). One can see that the larger the ratio z_0/y_0 , the greater the conversion efficiency $P_{\text{las}}^{\text{out}}/P_{0\text{p}}$. If at $z_0/y_0 = 5$ the maximum ratio $P_{\text{las}}^{\text{out}}/P_{0p}$ is 0.11 [curve (7)], then at $z_0/y_0 =$ 100 the maximum conversion efficiency reaches 0.53 [curve (I)]. With an increase in the ratio z_0/y_0 , the frequency detuning of the radiation generated by the laser, $arOmega_{
m las}^{
m max}$, increases, at which the conversion efficiency reaches its maximum value: $\Omega_{\text{las}}^{\text{max}}/(2\pi c) = -33 \text{ cm}^{-1}$ at $z_0/y_0 = 5$ [curve (7)] and $\Omega_{\text{las}}^{\text{max}}/(2\pi c) = -100 \text{ cm}^{-1} \text{ at } z_0/y_0 = 100 \text{ [curve (}1\text{)]}.$ The frequency of the laser radiation under normal conditions will be in the region of the gain band maximum (in the region of the frequency detuning $\Omega_{\text{las}}^{\text{max}}$). A distinctive feature of a two-level gas laser is that it has a wide gain band, which is the wider, the larger the ratio z_0/y_0 [compare curves (1-7) in Fig. 2]. This circumstance makes it possible to smoothly tune the generation frequency (when using optical intracavity frequency-selective elements).



Figure 2. Dependences of the conversion efficiency of pump radiation into laser radiation on the frequency detuning Ω_{las} at the optimal values of the parameter Ny_0 and the reflection coefficient R_0 of the output mirror, $I_{0p} = 3 \text{ kW cm}^{-2}$, $\Delta \omega / (2\pi c) = 1 \text{ cm}^{-1}$, T = 430 K, $p_{\text{He}} = 5 \text{ atm}$; $R_1 = R_p = 1$, $T_r = 0.995$, $\omega_p = \omega_{21}$ and different rations z_0/y_0 : (1) $z_0/y_0 = 100$, $R_0 = 0.956$, $Ny_0 = 3.03 \times 10^{14} \text{ cm}^{-2}$; (2) $z_0/y_0 = 70$, $R_0 = 0.962$, $Ny_0 = 3.29 \times 10^{14} \text{ cm}^{-2}$; (3) $z_0/y_0 = 50$, $R_0 = 0.967$, $Ny_0 = 3.55 \times 10^{14} \text{ cm}^{-2}$; (4) $z_0/y_0 = 30$, $R_0 = 0.974$, $Ny_0 = 3.97 \times 10^{14} \text{ cm}^{-2}$; (5) $z_0/y_0 = 20$, $R_0 = 0.979$, $Ny_0 = 4.33 \times 10^{14} \text{ cm}^{-2}$; (6) $z_0/y_0 = 10$, $R_0 = 0.9866$, $Ny_0 = 4.99 \times 10^{14} \text{ cm}^{-2}$; (7) $z_0/y_0 = 5$, $R_0 = 0.992$, $Ny_0 = 5.66 \times 10^{14} \text{ cm}^{-2}$.

Regarding the half-width of the emission spectrum of the pump diodes $[\Delta\omega/(2\pi c) = 1 \text{ cm}^{-1}]$, which we adopted in our calculations, we note that modern laser diodes used to pump alkali metal vapour lasers can have a half-width of the emission spectrum less than 1 cm⁻¹ [27]. In this case, the radiation power density of individual lines of laser diodes is 1 kW cm⁻² [27]. Therefore, to achieve the radiation intensity of the pump diodes, $I_p = 3 \text{ kW cm}^{-2}$, focusing of the diode radiation is required.

Figure 3 demonstrates the influence of the reflection coefficient R_0 of the output mirror on the efficiency of converting pump radiation into laser radiation. With a small deviation of the reflection coefficient R_0 from the optimal value [curve (2) in Fig. 3 corresponds to this value] in the direction of its decrease [curve (3)] or increase [curve (1)], the efficiency of the laser operation noticeably decreases.

Figure 4 shows the results of calculations of the effect of the buffer gas pressure p_{He} on the efficiency of conversion of pump radiation into laser radiation at optimal values of the parameter Ny_0 and optimal values of the reflection coefficient R_0 of the output mirror. Each value of p_{He} corresponds to its own optimal values of the parameter Ny_0 and the reflection coefficient R_0 of the output mirror, at which the maximum value of the conversion efficiency $(P_{\text{las}}^{\text{out}}/P_{0p})_{\text{max}}$. is achieved. One can see from Fig. 4 that the higher the buffer gas pressure, the higher the conversion efficiency $(P_{\text{las}}^{\text{out}}/P_{0p})_{\text{max}}$ and the



Figure 3. Dependences of the conversion efficiency $P_{\text{las}}^{\text{out}}/P_{0p}$ on the frequency detuning Ω_{las} at $I_{0p} = 3 \text{ kW cm}^{-2}$, $\Delta\omega/(2\pi c) = 1 \text{ cm}^{-1}$, T = 430 K, $Ny_0 = 3.55 \times 10^{14} \text{ cm}^{-2}$, $P_{\text{He}} = 5 \text{ atm}$, $z_0/y_0 = 50$, $R_1 = R_p = 1$, $T_r = 0.995$, $\omega_p = \omega_{21}$ and different reflection coefficients of the output mirror: $R_0 = (1) 0.985 (2) 0.967$, (3) 0.930.

wider the laser gain band. If, at $p_{\text{He}} = 0.5$ atm, the maximum conversion efficiency $(P_{\text{las}}^{\text{out}}/P_{0p})_{\text{max}} = 0.086$ and the gain bandwidth $\Delta \Omega_{\text{las}}/(2\pi c) = 35 \text{ cm}^{-1}$ [curve (6)], then at $p_{\text{He}} = 10$ atm, the maximum conversion efficiency $(P_{\text{las}}^{\text{out}}/P_{0p})_{\text{max}} = 0.46$ and the gain bandwidth $\Delta \Omega_{\text{las}}/(2\pi c) = 232 \text{ cm}^{-1}$ [curve (1)].



Figure 4. Dependences of the conversion efficiency $P_{\text{las}}^{\text{out}}/P_{0p}$ on the frequency detuning Ω_{las} at the optimal values of the parameter Ny_0 and the reflection coefficient R_0 of the output mirror, $I_{0p} = 3 \text{ kW cm}^{-2}$, $\Delta\omega/(2\pi c) = 1 \text{ cm}^{-1}$, T = 430 K, $z_0/y_0 = 50$, $R_1 = R_p = 1$, $T_r = 0.995$, $\omega_p = \omega_{21}$ and various pressures of the helium buffer gas: (1) $p_{\text{He}} = 10 \text{ atm}$, $R_0 = 0.965$, $Ny_0 = 3.64 \times 10^{14} \text{ cm}^{-2}$; (2) $p_{\text{He}} = 5 \text{ atm}$, $R_0 = 0.967$, $Ny_0 = 3.55 \times 10^{14} \text{ cm}^{-2}$; (3) $p_{\text{He}} = 3 \text{ atm}$, $R_0 = 0.970$, $Ny_0 = 3.61 \times 10^{14} \text{ cm}^{-2}$; (4) $p_{\text{He}} = 2 \text{ atm}$, $R_0 = 0.974$, $Ny_0 = 3.71 \times 10^{14} \text{ cm}^{-2}$; (5) $p_{\text{He}} = 1 \text{ atm}$, $R_0 = 0.983$, $Ny_0 = 3.91 \times 10^{14} \text{ cm}^{-2}$; (6) $p_{\text{He}} = 0.5 \text{ atm}$, $R_0 = 0.991$, $Ny_0 = 3.81 \times 10^{14} \text{ cm}^{-2}$.

[curve (1)].



Figure 5. Dependences of the conversion efficiency $P_{\text{las}}^{\text{out}}/P_{0p}$ on the frequency detuning Ω_{las} at the optimal values of the parameter Ny_0 and the reflection coefficien R_0 , $p_{\text{He}} = 5 \text{ arm}$, $\Delta \omega / (2\pi c) = 1 \text{ cm}^{-1}$, T = 430 K, $z_0/y_0 = 50$, $R_1 = R_p = 1$, $T_r = 0.995$, $\omega_p = \omega_{21}$ and different pump radiation intensities: (1) $I_{0p} = 1 \text{ kW cm}^{-2}$, $R_0 = 0.991$, $Ny_0 = 1.96 \times 10^{14} \text{ cm}^{-2}$; (2) $I_{0p} = 2 \text{ kW cm}^{-2}$, $R_0 = 0.978$, $Ny_0 = 2.89 \times 10^{14} \text{ cm}^{-2}$; (3) $I_{0p} = 3 \text{ kW cm}^{-2}$, $R_0 = 0.967$, $Ny_0 = 3.55 \times 10^{14} \text{ cm}^{-2}$; (4) $I_{0p} = 5 \text{ kW cm}^{-2}$, $R_0 = 0.950$, $Ny_0 = 4.51 \times 10^{14} \text{ cm}^{-2}$; (5) $I_{0p} = 10 \text{ kW cm}^{-2}$, $R_0 = 0.916$, $Ny_0 = 6.12 \times 10^{14} \text{ cm}^{-2}$.

The effect of the half-width of the pump radiation spectrum $\Delta\omega$ on the conversion efficiency is shown in Fig. 6. The efficiency decreases with increasing half-width of the pump radiation spectrum: for the parameters corresponding to Fig. 6, the maximum conversion efficiency is $(P_{\text{las}}^{\text{out}}/P_{0p})_{\text{max}} = 0.44$ at $\Delta\omega/(2\pi c) = 1 \text{ cm}^{-1}$ [curve (1)] and $(P_{\text{las}}^{\text{out}}/P_{0p})_{\text{max}} = 0.26$ at $\Delta\omega/(2\pi c) = 4 \text{ cm}^{-1}$ [curve (4)].

Let us pay attention to the following circumstance. Numerical analysis shows that for a given pump radiation intensity and optimal values of the parameters $(Ny_0)_{opt}$ and $(R_0)_{opt}$ at the same ratio of the homogeneous half-width of the absorption line to the half-width of the pump radiation spectrum $\Gamma/\Delta\omega$, the maximum efficiency of conversion of pump radiation into laser radiation $(P_{las}^{out}/P_{0p})_{max}$ is the same (Fig. 7). Thus, for curves (1-5) with the same ratio $\Gamma/\Delta\omega = 0.88$, the ratios $(P_{las}^{out}/P_{0p})_{max}$ equal to 0.40 are also the same, which are achieved at $\Omega_{las}^{max}/(2\pi c) = -170, -144, -102, -55$ and -29 cm^{-1} , respectively. The frequency detuning modulus $|\Omega_{las}^{max}|$ is approximately proportional to the homogeneous half-width of the absorption line Γ (or to the pressure of the helium buffer gas p_{He}).

The efficiency of conversion of pump radiation into laser radiation, $P_{\text{las}}^{\text{out}}/P_{0p}$, depends nonmonotonically on the param-



Figure 6. Dependences of conversion efficiency $P_{\rm las}^{\rm out}/P_{\rm 0p}$ on frequency detuning $\Omega_{\rm las}$ at optimal values of the parameter Ny_0 and reflection coefficient R_0 , $I_{\rm 0p} = 3$ kW cm⁻², $p_{\rm He} = 5$ atm, T = 430 K, $z_0/y_0 = 50$, $R_1 = R_p = 1$, $T_r = 0.995$, $\omega_p = \omega_{21}$ and different half-widths of the pump radiation spectrum $\Delta\omega$: (1) $\Delta\omega/(2\pi c) = 1$ cm⁻¹, $R_0 = 0.967$, $Ny_0 = 3.55 \times 10^{14}$ cm⁻²; (2) $\Delta\omega/(2\pi c) = 2$ cm⁻¹, $R_0 = 0.972$, $Ny_0 = 3.80 \times 10^{14}$ cm⁻²; (3) $\Delta\omega/(2\pi c) = 3$ cm⁻¹, $R_0 = 0.976$, $Ny_0 = 4.02 \times 10^{14}$ cm⁻²; (4) $\Delta\omega/(2\pi c) = 4$ cm⁻¹, $R_0 = 0.980$, $Ny_0 = 4.20 \times 10^{14}$ cm⁻².



Figure 7. Dependences of the conversion efficiency P_{1as}^{out}/P_{0p} on the frequency detuning Ω_{las} at a constant ratio $\Gamma/\Delta\omega = 0.88$, optimal values of Ny_0 and R_0 , $I_{0p} = 3$ kW cm⁻², T = 430 K, $z_0/y_0 = 50$, $R_0 = 0.97$, $R_1 = R_p = 1$, $T_r = 0.995$, $\omega_p = \omega_{21}$ and various buffer gas pressures: (1) $p_{He} = 12$ atm, $\Delta\omega/(2\pi c) = 4$ cm⁻¹, $Ny_0 = 4.05 \times 10^{14}$ cm⁻²; (2) $p_{He} = 9$ atm, $\Delta\omega/(2\pi c) = 3$ cm⁻¹, $Ny_0 = 3.95 \times 10^{14}$ cm⁻²; (3) $p_{He} = 6$ atm, $\Delta\omega/(2\pi c) = 2$ cm⁻¹, $Ny_0 = 3.78 \times 10^{14}$ cm⁻²; (4) $p_{He} = 3$ atm, $\Delta\omega/(2\pi c) = 1$ cm⁻¹, $Ny_0 = 3.61 \times 10^{14}$ cm⁻²; (5) $p_{He} = 1.5$ atm, $\Delta\omega/(2\pi c) = 0.5$ cm⁻¹, $Ny_0 = 3.52 \times 10^{14}$ cm⁻².

eter Ny_0 and the reflection coefficient R_0 of the output mirror and reaches a maximum value $(P_{\text{las}}^{\text{out}}/P_{0p})_{\text{max}}$ at some optimal parameter $(Ny_0)_{\text{opt}}$ and some optimal reflection coefficient $(R_0)_{\text{opt}}$. Figure 8 shows the results of calculations of the quantities $(P_{\text{las}}^{\text{out}}/P_{0p})_{\text{max}}$, $(R_0)_{\text{opt}}$, and $(Ny_0)_{\text{opt}}$ as functions of the pump radiation intensity I_{0p} at buffer gas pressures $p_{He} = 2, 3$, and 10 atm. One can see from Fig. 8a that the maximum conversion efficiency increases monotonically with increasing pump radiation intensity and buffer gas pressure. The optimal reflection coefficient $(R_0)_{opt}$ of the output mirror decreases monotonically with increasing I_{0p} and p_{He} (Fig. 8b). In this case, the optimal parameter $(Ny_0)_{opt}$ increases monotonically with increasing pump radiation intensity and depends weakly on the buffer gas pressure (Fig. 8c).

Figure 9 shows the dependences $I_{\text{las}}^{\text{out}}(y)$ for various detunings of the laser radiation frequency Ω_{las} . The calculation was performed with parameters corresponding to curve (3) in Fig. 4 and curve (4) in Fig. 7. When calculating curve (1) in Fig. 9, we set the frequency detuning $\Omega_{\text{las}} = \Omega_{\text{las}}^{\text{max}}$, at which the conversion efficiency reaches its maximum value $(P_{\text{las}}^{\text{out}}/P_{0p})_{\text{max}}$. When calculating curve (2), two frequency detunings were specified, at which the conversion efficiency was 50% of its maximum value. It can be seen that the intensity of the laser radiation decreases faster with increasing y at the maximum of the gain band than at the edges of the band.

A two-level gas laser with transverse diode pumping is capable of generating cw optical radiation with a sufficiently high power. Let us find the average specific output of radiation power from the active medium, $P_{\text{las}}^{\text{out}}/V$, using the relation

$$\frac{P_{\text{las}}^{\text{out}}}{V} = \frac{P_{\text{las}}^{\text{out}} I_{0p}}{P_{0p}} \frac{I_{0p}}{y_0}.$$
(55)

Let caesium atoms be the active medium in the laser cell, and helium be used as a buffer gas. In this case, the radiation from the pump diodes at the cell input has the following characteristics: $I_{0p} = 3 \text{ kW cm}^{-2}$, $\Delta\omega/(2\pi c) = 1 \text{ cm}^{-1}$, and $\omega_p = \omega_{21}$. We assume that $z_0/y_0 = 50$, $p_{\text{He}} =$ 5 atm, gas mixture temperature T = 430 K, reflectances of the mirrors $R_p = 1$ and $R_1 = 1$, and effective transmittance $T_{\rm r} = 0.995$. Under these conditions, the maximum value of $P_{\rm las}^{\rm out}/P_{\rm 0p}$ is equal to 0.44 [with frequency detuning $\Delta\omega/(2\pi c)$ = -79 cm⁻¹], $(Ny_0)_{\rm opt}$ = 3.55 × 10¹⁴ cm⁻² and $(R_0)_{\rm opt}$ = 0.967 [curves (2) in Figs 3 and 4]. The cell width y_0 can be easily found from the value of the parameter Ny_0 . Let the concentration N of caesium atoms inside the cell be given by the temperature T = 430 K. At this temperature, $N = 3 \times 10^{14}$ cm⁻³ [28], and an optical value of $(Ny_0)_{opt} = 3.55 \times 10^{14}$ cm⁻³ is reached at $y_0 = 1.2$ cm. Using the above parameters of the working medium and pump radiation, we find from formula (55) that the average specific output of laser radiation power from the active medium, $P_{\text{las}}^{\text{out}}/V$ is approximately 1 kW cm⁻³. For cell sizes $z_0 = 60$ cm, $y_0 = 1.2$ cm, and $x_0 =$ 1.4 cm, the output power of laser radiation in the cw regime reaches 100 kW.

6. Conclusions

A new type of a gas laser, a two-level gas laser with transverse diode pumping, has been theoretically investigated. Laser radiation is generated without population inversion in the red wing of the spectral line of a system of two-level atoms upon resonant absorption of broadband radiation from pump diodes by active particles in a high-pressure buffer gas atmosphere. The reason for the occurrence of this effect is the fact that at high pressures of the buffer gas (when the homogeneous broadening due to the interaction of particles with the



Figure 8. (a) Dependences of the maximum conversion efficiency $(P_{las}^{out}/P_{0p})_{max}$ on the pump radiation intensity I_{0p} at the optimal parameters $(Ny_{0})_{opt}$ and reflection coefficients $(R_{0})_{opt}$ of the output mirror, (b) dependences of the optimal reflection coefficient $(R_{0})_{opt}$ on the radiation intensity I_{0p} at optimal parameters $(Ny_{0})_{opt}$, and (c) dependences of the optimal parameters $(Ny_{0})_{opt}$, and (c) dependences of the optimal parameters $(Ny_{0})_{opt}$ on I_{0p} at optimal reflection coefficients $(R_{0})_{opt}$. Calculation parameters are as follows: $\Delta\omega/(2\pi c) = 1 \text{ cm}^{-1}$, $z_{0}/y_{0} = 50$, T = 430 K, $R_{1} = R_{p} = 1$, $T_{r} = 0.995$, $\omega_{p} = \omega_{2}$, and $p_{\text{He}} = (1) 2$, (2) 3 and (3) 10 atm.



Figure 9. Dependences of the laser radiation intensity $I_{\text{las}}^{\text{out}}$ on the *y* coordinate at optimal values of the parameter Ny_0 and the reflection coefficient R_0 of the output mirror, $I_{0p} = 3 \text{ kW cm}^{-2}$, $\Delta\omega/(2\pi c) = 1 \text{ cm}^{-1}$, $p_{\text{He}} = 3 \text{ atm}$, T = 430 K, $Ny_0 = 3.61 \times 10^{14} \text{ cm}^{-2}$, $z_0/y_0 = 50$, $R_0 = 0.970$, $R_1 = R_p = 1$, $T_r = 0.995$, $\omega_p = \omega_{21}$ and frequency detunings $\Omega_{\text{las}}/(2\pi c) = (1)$ -55.0, -12.8 and -95.5 cm⁻¹ (2).

buffer gas significantly exceeds the natural one) in the red wing of the spectral line, the probability of stimulated emission exceeds the probability of absorption (the spectral densities of the Einstein coefficients for absorption and stimulated emissions due to collisions cease to be equal to each other).

The operation of a two-level laser is described by a complex system of differential equations, which in the general case can be solved only with the help of numerical methods. For a not too small reflection coefficient R_0 of the output mirror, the populations of the levels of the active medium atoms are practically independent of the *z* coordinate along the cell axis. In this approximation, the system of differential equations is greatly simplified and admits an analytical solution, which makes it possible to exhaustively determine any laser energy characteristics.

Calculations using analytical formulas show that the efficiency of conversion of pump radiation into laser radiation, $P_{\text{last}}^{\text{out}}/P_{0p}$, is the greater, the longer the active medium, the higher the buffer gas pressure and the intensity of the pump radiation, and the smaller the width of the pump radiation spectrum. There exist optimal values of the parameter Ny_0 (the number of active atoms in a cell in a gas column of height y_0 with a unit cross section) and the reflection coefficient R_0 of the output mirror, at which the maximum of the ratio $P_{\text{las}}^{\text{out}}/P_{0p}$ as a function of the detuning of the laser-generated radiation frequency Ω_{las} has the highest value.

With realistic parameters of the working medium and pump radiation in a sufficiently long active medium (with a cell length-to-width ratio $z_0/y_0 = 50$), the conversion efficiency reaches 44% at a buffer gas pressure of 5 atm, a pump diode radiation intensity of 3 kW cm⁻², and a half-width pump radiation spectrum of 1 cm⁻¹. In this case, the average specific output of laser radiation power from the active medium is approximately 1 kW cm⁻³. A two-level gas laser with transverse diode pumping is capable of generating continuous optical radiation with a very high (up to 100 kW) power. It should be noted that since a two-level gas laser has a fairly wide gain band (several tens of cm^{-1}), it is possible to obtain frequency-tunable lasing in a selective resonator (using optical intracavity frequency-selective elements).

Acknowledgements. The study was carried out at the expense of subsidies for the financial support for fulfilling the State Task (Project No. AAAA-A21-121021800168-4) at the Institute of Automation and Electrometry of the Siberian Branch of the Russian Academy of Sciences.

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