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Magnetic coherence effects in the spectroscopy of transitions between energy levels with total angular momenta *J* = 1/2 and *J* = 1 using unidirectional waves

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Abstract. **The physical processes that form the spectra of saturated absorption resonances at atomic transitions between energy levels with angular momenta $J = 1/2$ and $J = 1$ in the field of two unidirectional linearly polarised laser waves are studied analytically and numerically. It is shown that the specific features of the resonance spectra are formed in the Λ-schemes of transitions and manifest themselves in the form of narrow coherent structures – dips due to the magnetic coherence (optical orientation) of the transition levels induced by the optical fields. In this case, the energy levels of the lower state make the main contribution, and the contribution of the transfer of magnetic coherence from the sublevels of the upper state to the lower ones in the resonance amplitude manifests itself as an additive. Conditions are found under which the nonlinear resonance is exclusively coherent. The effect of the saturating wave field on the shape of coherent resonance structures is studied.**

Keywords: *saturated absorption resonance, unidirectional waves, closed and open transitions, magnetic coherence of energy levels.*

1. Introduction

Studies of nonlinear spectroscopic effects under the resonant interaction of several light fields with degenerate atomic transitions have been carried out for a long time. Interest in transitions induced by radiation was indicated as early as in the works of A. Einstein (1916) and S.I. Vavilov (1918). They both considered incoherent broadband radiation that changes the level populations. Now the saturation effect, as a rule, always accompanies spectroscopic studies using laser radiation. Note that the appearance of coherence of atomic states in two-photon transitions was also first discovered in the prelaser era using a source of incoherent radiation [1]. With the advent of lasers, the field of study of such coherent phenomena expanded considerably by 1972 (see, e.g., review [2]) and continues to grow. Interest in the topic is supported by an increase in the number of experimental techniques, including such complex ones as the study of cold atoms. Subsequently, the resonances due to the coherence of atomic states in the presence of laser radiation were called electromagnetically induced transparency (EIT) and electromagnetically induced absorption (EIA) resonances. Having arisen in the process of studying nonlinear optical phenomena under the interaction of laser radiation with gaseous media, the scope of these effects has extended to many other systems, the practical applications of which are expected. However, here, too, the need to obtain accurate analytical solutions and interpret the observed phenomena is associated with nonlinear spectroscopy of gaseous media and has stimulated work in this area, including numerical simulation of experiments as a method that makes it possible to study situations that are not always achievable experimentally. Many of the phenomena discovered in the 1970s were 'rediscovered' and renamed in papers on EIT and EIA. We drew attention to this, as well as to misconceptions in the interpretation of a number of results obtained at that time, in the Introduction to Ref. [3].

An important example of coherent phenomena in transitions from the ground state of alkali metal atoms are EIT resonances [4], which are based on the phenomenon of coherent population trapping (CPT) [5], as well as resonances of the opposite sign – the EIA resonances, first found in Ref. [6]. The appearance of these EIA resonances was explained in Ref. [7] by the effect of spontaneous transfer of the magnetic coherence (MC) of the excited state atomic levels to the ground state, for which the regularities of manifestation in saturated absorption spectroscopy were first considered in Ref. [8]. However, the anomalies of EIA resonances recorded later in the experiments [9, 10] could not be explained within the framework of the mechanism of Ref. [7]. In this regard, to explain these effects, other processes, such as optical pumping and CPT [9], collisions [11], which seemingly resemble experimental resonance structures, were considered, and not always justifiably, as well. Nevertheless, in Ref. [12], developing the concept [7], it was stated that the main mechanism for the formation of EIA resonances in closed transitions with any value of the total angular momenta of the levels is precisely the spontaneous transfer of the MC from the sublevels of the upper state to those of the lower state. However, later our studies showed that in the formation of structures of nonlinear resonances, including EIA resonances, both in simple and in degenerate transitions (between energy levels with angular momenta $J = 1, 2$), the spontaneous transfer of the MC of the upper state levels is not of the primary importance.

For example, it was shown in Ref. [13] that in a system of two nondegenerate energy levels, the narrow structure of a nonlinear resonance in the field of two unidirectional waves coupled to an open transition manifests itself as an EIT resonance, and in a closed transition, as an EIA resonance. The reason for the appearance of these structures is the coherent beats of the populations of the transition levels in a double-

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frequency field [14, 15]. In the case of transitions between energy levels with $J = 1$, as established in Ref. [16], narrow structures of saturated absorption resonances are formed in Λ schemes of transitions and, for parallel polarisations of the fields, are determined by coherent beats of the energy level populations [13], whereas for orthogonal polarisations, they are determined by the nonlinear interference effect (NIEF) [14]. In this case, even when the upper state is closed, two-level transitions between magnetic sublevels turn out to be open, which, according to Ref. [13], gives rise to a narrow structure in the form of an EIT resonance. The effect of magnetic coherence of levels forms EIT and EIA resonances in magnetic scanning spectra. In this case, the main contribution is made by the MC of the lower state energy levels, while the contribution of the spontaneous transfer of the MC from the levels of the upper state to the lower state, postulated in Refs [10, 12] as the major one, is small and manifests itself only as an additive. The results of Ref. [16] are also valid for $J \to J$ and $J \to J-1$ transitions, since in these transitions the spectra of nonlinear resonances are also formed in open Λ configurations.

A different situation arises for transitions of the $J \to J+1$ type [17, 18], where, due to the difference in the oscillator strengths of transitions between magnetic sublevels, the nonlinear resonance spectrum is mainly due to sublevels with the maximum magnetic number M forming V diagrams of transitions. Exactly in V diagrams, closed two-level transitions are realised, in which the form of narrow resonance structures radically depends on the degree of openness of the atomic transition [13]. The effect of spontaneous transfer of the MC of the levels for transitions of this type also does not affect the qualitative form of the narrow resonance structure. In this case, it was found that a change in the intensity of the probe field could change the type of narrow resonance (from EIT to EIA and vice versa) [17].

Note that the main features of the nonlinear resonance formation in the resonant interaction of a double-frequency field with degenerate atomic transitions [16–18] were found by solving the problem numerically, since the analytical solutions for transitions between levels with angular momenta $J > 1$ are complicated and difficult to analyse. In this regard, of interest are the simplest degenerate transitions (between levels with angular momenta $J = 1/2$ and $J = 1$), for which analytical solutions are possible. These solutions allow one to establish quantitative relationships between the processes that form the resonance spectrum of saturated absorption in the probe field method, including the contributions of the MC induced by the optical fields and its transfer from the upper state to the lower one. The obtained relations are important for determining the contributions of these processes in atomic systems of higher complexity, including for clarifying the mechanism of the EIA resonance formation in Ref. [6].

2. Probe field absorption spectrum in a system of degenerate levels with total electron angular momenta $J = 1/2$ and $J = 1$

Let us consider a problem of the probe field absorption spectrum in atomic transitions between the energy levels with the total angular momenta $J = 1/2$ and $J = 1$ in the presence of a strong radiation field. The diagrams of levels and transitions are shown in Figs 1 and 2. The strong wave is assumed to be plane monochromatic, linearly polarised (frequency ω, wave vector $\boldsymbol{k}$, electric field strength $\boldsymbol{E}$) and resonant to the atomic transition $m-n$ (transition frequency ω_{mn}). The probe wave is also monochromatic (frequency ω_μ, wave vector $\boldsymbol{k}_\mu$, electric field strength $\boldsymbol{E}_\mu$), polarised orthogonally to the polarisation of the strong field, not weak, and directed along the propagation direction of the strong wave. The gas is assumed to be rarefied enough to neglect collisions.

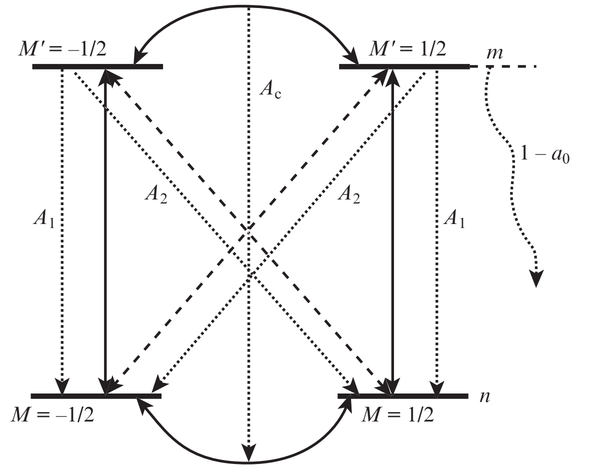

Figure 1. Schematic of kinetic processes for the transition between levels with angular momenta $J = 1/2$. Solid and dashed arrows denote transitions under the action of strong and probe fields, respectively; dotted arrows denote spontaneous transitions (rates A_1, A_2, and A_c); solid arc arrows denote magnetic coherences; and dotted wavy arrows denote spontaneous decay of magnetic sublevels of state m into levels of state n and other lower levels; $1 - a_0$ is the fraction of this process, and $a_0 = A_{mn}/\Gamma_m$ is the branching parameter of the transition.

When solving the problem, we will proceed from the kinetic equations for the density matrix of the atomic system within the model of relaxation constants [14]. In this case, the dynamics of diagonal ($\rho_i = \rho_{ii}$) and off-diagonal (ρ_{ik}) elements of the density matrix is determined by the system of equations:

$$\frac{\mathrm{d}\rho_i}{\mathrm{d}t} + \Gamma_i\rho_i = Q_i + \sum_k A_{ki}\rho_k - 2\,\mathrm{Re}\Big(\mathrm{i}\sum_j V_{ij}\rho_{ji}\Big), \tag{1}$$

$$\frac{\mathrm{d}\rho_{ik}}{\mathrm{d}t} + (\Gamma_{ik} + \mathrm{i}\omega_{ik})\rho_{ik} = -\mathrm{i}[V,\rho]_{ik} + R^{\mathrm{s}}_{ik}, \tag{2}$$

where $\mathrm{d}/\mathrm{d}t = \partial/\partial t + \boldsymbol{v}\nabla$ is the total derivative operator; $\boldsymbol{v}$ is the atomic velocity vector; Γ_i are the relaxation constants of the energy levels; Γ_{ik} are the coherence relaxation constants for allowed ($\Gamma_{ik} = \Gamma$) and forbidden ($\Gamma_{ik} = \Gamma_m$, Γ_n) transitions between magnetic sublevels of the states m and n; Q_i are the excitation rates of these sublevels, which determine their populations in the absence of light fields (in the case of the ground state, Γ_n is determined by the average flight time in the atom–field interaction region); $V = -G\exp[\mathrm{i}(\boldsymbol{kr} - \omega t)] - G^\mu\exp[\mathrm{i}(\boldsymbol{k}_\mu\boldsymbol{r} - \omega_\mu t)] + \mathrm{h.c.}$ is the operator of the atom interaction with pump and probe fields; $G = dE/2\hbar$, $G^\mu = dE_\mu/2\hbar$; and d is the transition dipole moment operator. The terms $A_{ki}\rho_k$ in the system of Eqns (1) determine the contributions of the spontaneous decay of the kth sublevel of the upper state m to the ith sublevel of the lower state n (the rate of this process is determined by the Einstein coefficient A_{mn}), they are present

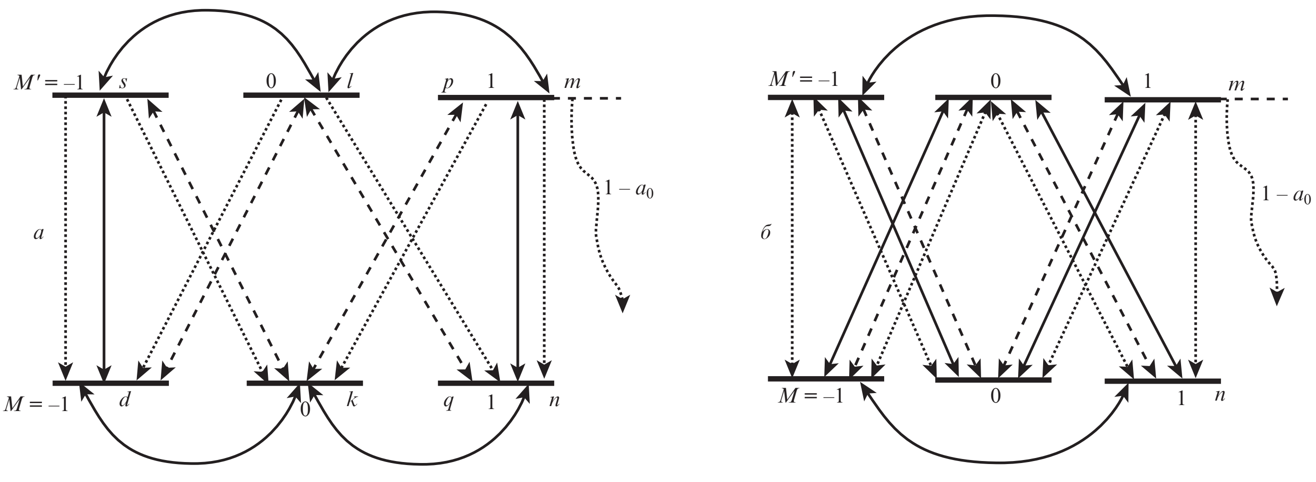

Figure 2. Schematic of the interaction of optical fields with sublevels of the transition $J = 1 \to J = 1$, when the wave propagates (a) orthogonally and (b) parallel to the direction of the magnetic field. Solid lines denote the strong field, dashed lines denote the probe field, dotted lines (straight and wavy) indicate spontaneous decay of magnetic sublevels of state m into levels of state n and other lower levels; $1 - a_0$ is the fraction of this process.

in the equations for the populations of the lower energy levels and are absent in the equations for the populations of the upper levels. The term R^{s}_{ik} in the system of Eqns (2) describes the spontaneous transfer of the magnetic coherence of the sublevels of the upper state to the sublevels of the lower state at a rate A_c determined by the Einstein coefficient A_{mn} of the transition (see below).

Let us consider the problem of the interaction of optical fields with the above transitions in a coordinate system with the quantisation axis along the direction of the strong field electric field vector $\boldsymbol{E}$ (directed along the z axis). Then the strong field induces transitions between magnetic sublevels (Fig. 1, Fig. 2a) with a change in the magnetic quantum number $M - M' = 0$, whereas the orthogonally polarised probe field induces transitions with $M - M' = \pm 1$. In this case, optical fields induce low-frequency coherence between the magnetic sublevels of the upper and lower states with $M - M' = \pm 1$ (the so-called optical orientation), whose contribution to the probe wave absorption spectrum will be of further interest. It is in this formulation that one can obtain simple analytical solutions of the problem for these transitions. This consideration is also valid for the problem of waves propagating through a medium orthogonally to a weak external magnetic field.

When considering the problem in a coordinate system with the quantisation axis along the direction of the strong field wave vector $\boldsymbol{k}$ ($\boldsymbol{k}$ is directed along the z axis; the case of light waves propagating along the direction of the magnetic field), both light fields are coupled to the same magnetic sublevels, inducing transitions with $M - M' = \pm 1$. In this case, the magnetic coherence is absent for the transition between the levels with momenta $J = 1/2$, whereas for the transition between the levels with momenta $J = 1$, the magnetic coherence is induced between the sublevels of each of the states with a change in the magnetic quantum number $M - M' = \pm 2$ (the so-called optical alignment, Fig. 2b). This effect is produced both by the individual action of the strong or probe wave and by their joint action. In this formulation of the problem, a large number of independent parameters and the complexity of expressions make the analytical solutions of Eqns (1) and (2) practically useless and numerical solutions are more informative. In particular, the numerical solutions of the equations in Refs [16, 17] made it possible to determine the physical processes giving rise to narrow structures in the spectra of saturated absorption resonances for a number of degenerate atomic transitions, including the transition between levels with angular momenta $J = 1$, and to reveal the role of optical alignment in the formation of these narrow structures.

2.1. Probe field absorption spectrum in a system of two levels with angular momenta $J = 1/2$

Consider a transition between the energy levels with angular momenta $J = 1/2$ (Fig. 1) in the coordinate system with the quantisation axis directed along the strong wave electric field vector $\boldsymbol{E}$. We seek the solutions of Eqns (1) and (2) according to the procedure of the probe field method [14, 19], up to terms linear in G^{μ}, in the form

$$\rho_{mi} = \rho^{0}_{mi} + r_{mi}\exp(-\mathrm{i}\varepsilon t) + r^{*}_{mi}\exp(\mathrm{i}\varepsilon t),$$

$$\rho_{nk} = \rho^{0}_{nk} + r_{nk}\exp(-\mathrm{i}\varepsilon t) + r^{*}_{nk}\exp(\mathrm{i}\varepsilon t), \qquad (3)$$

$$\rho_{ik} = \rho^{0}_{ik}(-\mathrm{i}\Omega t) + r_{ik}\exp[-\mathrm{i}(\Omega + \varepsilon)t] + r^{1}_{ik}\exp[-\mathrm{i}(\Omega - \varepsilon)t]$$

or

$$\rho_{ik} = r_{jik}\exp(-\mathrm{i}\varepsilon t) + r^{1}_{jik}\exp(\mathrm{i}\varepsilon t),$$

where $j = m, n$; $i = +1/2, -1/2$; $k = -1/2, +1/2$; and $\varepsilon = \omega_{\mu} - \omega = \Omega_{\mu} - \Omega$. For immobile atoms $\Omega = \omega - \omega_{mn}$ and $\Omega_{\mu} = \omega_{\mu} - \omega_{mn}$. Taking the motion of atoms into account is reduced to the replacemen $\Omega \to \Omega - \boldsymbol{k}\boldsymbol{v}$, $\Omega_{\mu} \to \Omega_{\mu} - \boldsymbol{k}_{\mu}\boldsymbol{v}$, and $\varepsilon \to \varepsilon = \omega_{\mu} - \omega - (\boldsymbol{k}_{\mu} - \boldsymbol{k})\boldsymbol{v}$ in the equations, where $\boldsymbol{v}$ is the atomic velocity vector.

The first two solutions (3) describe the populations of the sublevels of the states m and n, and the third one describes the medium polarisation contributed by the allowed transitions (optical coherence) between the sublevels of the states m and n. The fourth solution describes the medium polarisation introduced by forbidden transitions between the sublevels of one state (the MC of the levels).

For the system of levels under consideration (see Fig. 1), we have the following relationships between the rates of spontaneous decay of the magnetic sublevels A_1, A_2 and the magnetic coherence A_c: $A_1 = A_{mn}/3$, $A_2 = 2A_{mn}/3$, $A_1 + A_2 = A_{mn}$, and $A_c = -A_{mn}/3$, where A_{mn} is the first Einstein coefficient of the transition [14, 19]. The matrix elements of the operator of interaction with the field for the transition between magnetic sublevels with $M' \to M$ are $G_{M',M} = G_{\pm 1/2, \pm 1/2}$, which we denote as $G_{+1/2,+1/2} = G_+$, $G_{-1/2,-1/2} = G_-$, $G_{+1/2,-1/2} = G_{+-}$, and $G_{-1/2,+1/2} = G_{-+}$. For these elements, the following relations are valid: $G_+ = -G_-$ and $G_{+-} = -G_{-+}$.

Let us also specify the elements of the density matrix in solution (3) and denote the coefficients at the populations of the magnetic sublevels of the states m and n as $\rho^0_{mi} = \rho^0_{m\pm}$, $r_{mi} = r_{m\pm}$, $\rho^0_{ni} = \rho^0_{n\pm}$, and $r_{ni} = r_{n\pm}$. We also denote the coefficients at the coherences for the allowed transitions between sublevels of states m and n as $\rho^0_{ik} = \rho^0_{mn\pm}$, $r_{ik} = r_{\pm\mp}$, and $r^1_{ik} = r^1_{\pm\mp}$; and the coefficients at the coherences for the transitions between sublevels of one state as $r_{jik} = r_{j\pm\mp}$ and $r^1_{jik} \to r^1_{\pm\mp}$ ($j = m, n$).

When solving Eqns (1) and (2) by the probe field method for $G \gg G^\mu$, the system of equations in the zero-order approximation with respect to G^μ for populations and coherence for the transition between magnetic sublevels with $M = M' = 1/2$ will have the form:

$$\Gamma_m \rho^0_{m+} + 2\,\mathrm{Re}(\mathrm{i}G^*_+ \rho^0_{mn+}) = Q_{m+},$$

$$\Gamma_n \rho^0_{n+} - A_1 \rho^0_{m+} - A_2 \rho^0_{m-} - 2\,\mathrm{Re}(\mathrm{i}G^*_+ \rho^0_{mn+}) = Q_{n+}, \qquad (4)$$

$$(\Gamma - \mathrm{i}\Omega)\rho^0_{mn+} + \mathrm{i}G_+(\rho^0_{m+} - \rho^0_{n+}) = 0.$$

The system of equations for respective quantities for the transition between magnetic sublevels with $M = M' = -1/2$ has a similar form.

Due to the symmetry of the problem with respect to the signs of the magnetic quantum numbers of states and the relation $A_1 + A_2 = A_{mn}$, the solutions to Eqns (4) are the same as for the two-level system [14]:

$$\rho^0_{m+} = N_m + \left(\frac{N_{nm}}{\Gamma_m T_{mn}} \frac{\kappa \Gamma^2}{\Gamma_s^2 + \Omega^2}\right),$$

$$\rho^0_{n+} - \rho^0_{m+} = N_{nm}\left(1 - \frac{\kappa\Gamma^2}{\Gamma_s^2 + \Omega^2}\right), \qquad (5)$$

$$\rho^0_{mn\pm} = \mathrm{i}G_\pm(\rho^0_{n+} - \rho^0_{m+})/(\Gamma - \mathrm{i}\Omega), \quad \rho^0_{m+} = \rho^0_{m-},$$

$$\rho^0_{n+} = \rho^0_{n-}, \; N_{nm} = N_n - N_m, \qquad (6)$$

where $\Gamma_s = \Gamma\sqrt{1+\kappa}$; $\kappa = 2|G_+|^2 \gamma_{mn}/\Gamma\Gamma_n\Gamma_m$; $\gamma_{nm} = \Gamma_m + \Gamma_n - A_{mn}$; and $N_n = Q_n/\Gamma_n$, and $N_m = Q_m/\Gamma_m$ are the populations of the magnetic sublevels of the states n and m in the absence of the strong field.

The systems of equations of the first-order approximation with respect to G^μ from Eqns (1) and (2), which determine the probe field absorption spectrum, are formed by the coefficients of the density matrix elements r_{+-}, r^1_{+-}, r_{-+}, and r^1_{-+} for allowed transitions and the coefficients r_{j+-}, r^1_{j+-}, r_{j-+}, and r^1_{j-+} ($j = m, n$) for transitions between sublevels within each of the states. In the case of a transition with $M' = 1/2 \to M = -1/2$, this system of equations has the form:

$$(\Gamma_m - \mathrm{i}\varepsilon) r_{m+-} + \mathrm{i}G^*_- r_{+-} - \mathrm{i}G_+ r^1_{+-} = \mathrm{i}G^\mu_{+-}\rho^0_{nm-},$$

$$(\Gamma_n - \mathrm{i}\varepsilon) r_{n+-} - A_c r_{m+-} - \mathrm{i}G^*_+ r_{+-} + \mathrm{i}G_- r^1_{+-} = -\mathrm{i}G^\mu_{+-}\rho^0_{nm+},$$

$$(p - \mathrm{i}\varepsilon) r_{+-} - \mathrm{i}G_+ r_{n+-} + \mathrm{i}G_- r_{m+-} = -\mathrm{i}G^\mu_{+-}(\rho^0_{m+} - \rho^0_{n-}), \qquad (7)$$

$$(p^* - \mathrm{i}\varepsilon) r^1_{+-} + \mathrm{i}G^*_- r_{n+-} - \mathrm{i}G^*_+ r_{m+-} = 0,$$

where $p = \Gamma - \mathrm{i}\Omega$.

The term $A_c r_{m+-}$ in the second equation of system (7) describes the effect of transferring the MC of the upper state levels to the lower state. Equations for the coefficients of the density matrix for the transition with $M' = -1/2 \to M = 1/2$ are obtained from equations (7) by replacing the signs in the indices, $+ \leftrightarrow -$. Both systems of equations are closed and uniquely solvable with respect to the coefficients. Solutions of Eqns (7) yield the expression of the coefficient r_{+-}:

$$r_{+-} = \mathrm{i}G^\mu_{+-}\frac{N_{nm}}{\Gamma - \mathrm{i}\Omega_\mu}\left(1 - \frac{\kappa\Gamma^2}{\Gamma_s^2 + \Omega^2}\right)[1 - J_+(\varepsilon)], \qquad (8)$$

$$J_+(\varepsilon) = \frac{|G_+|^2}{\Delta_+}(2\Gamma - \mathrm{i}\varepsilon)(\Gamma_m + \Gamma_n + A_c - 2\mathrm{i}\varepsilon)\frac{\Gamma - \mathrm{i}(\varepsilon - \Omega)}{\Gamma + \mathrm{i}\Omega}, \qquad (9)$$

where

$$\Delta_+ = [\Gamma - \mathrm{i}(\varepsilon + \Omega)][\Gamma - \mathrm{i}(\varepsilon - \Omega)](\Gamma_m - \mathrm{i}\varepsilon)(\Gamma_n - \mathrm{i}\varepsilon)$$
$$+ 2(\Gamma - \mathrm{i}\varepsilon)(\Gamma_m + \Gamma_n + A_c - 2\mathrm{i}\varepsilon)|G_+|^2.$$

The solution for the coefficient r_{-+} is similar to expressions (8) and (9) with the change of signs, $+ \leftrightarrow -$, in the subscripts.

According to Ref. [14], the absorption spectrum of the probe field is determined through the work of the field

$$P_\mu = -2\hbar\omega_\mu \mathrm{Re}\langle \mathrm{i}(r_{+-}G^{\mu *}_{+-} + r_{-+}G^{\mu *}_{-+})\rangle.$$

Using solution (8), we arrive at the expression for the work of the probe field:

$$P_\mu = 4\hbar\omega_\mu |G^\mu_{+-}|^2 \mathrm{Re}\left\{\frac{N_{nm}}{\Gamma - \mathrm{i}\Omega_\mu}\left(1 - \frac{\kappa\Gamma^2}{\Gamma_s^2 + \Omega^2}\right)[1 - J_+(\varepsilon)]\right\}, \qquad (10)$$

where it is taken into account that $|G^\mu_{+-}|^2 = |G^\mu_{-+}|^2$ and $|G_+|^2 = |G_-|^2$.

Expression (10) describes the absorption spectrum of the probe field for the transition between levels with angular momenta $J = 1/2$. The spectrum is formed by the effect of saturation of the level populations (incoherent process) and by coherent processes, such as the effect of level splitting by a strong field and nonlinear interference effects (NIEF) [14], caused by the MC and its transfer from the upper state to the lower one.

In the case of immobile atoms, $\Gamma_m \gg \Gamma_n$, in a weak saturating field ($\kappa \ll 1$), the work of the probe field near the centre of the line ($\varepsilon \ll \Gamma, \Gamma_m$; $\Omega = 0$) is derived from Eqn (10) as

$$P_\mu = 4\hbar\omega_\mu |G^\mu_{+-}|^2$$
$$\times \mathrm{Re}\left\{\frac{\delta N_{nm}}{\Gamma - \mathrm{i}\varepsilon}\left[1 - \frac{2|G_+|^2}{\Gamma\Gamma_m}\left(\frac{\Gamma_m - A_c}{\Gamma_m} + \frac{\Gamma_m + A_c - \Gamma_n}{\Gamma_n - \mathrm{i}\varepsilon}\right)\right]\right\}, \qquad (11)$$

where

$$\delta N_{nm} = N_{nm}\left(1 - \frac{\kappa\Gamma^2}{\Gamma_s^2 + \Omega^2}\right).$$

From Eqn (11) it follows that in the probe wave absorption line with half-width Γ, a narrow Lorentz dip with a half-width Γ_n and an amplitude determined by the factor $S = \Gamma_m - \Gamma_n + A_c$ is formed. Since for the transitions between energy levels with angular momenta $J = 1/2$ we have $A_c = -A_{mn}/3$ (see above), at $\Gamma_m \gg \Gamma_n$, the relation $S \approx \Gamma_m - \Gamma_n + A_c/3 > 0$ always holds, and the structure for any transitions appears as a narrow dip. The narrow structure in the nonlinear resonance shape (11) is due to the MC of the transition levels induced by optical the fields in transitions between the magnetic sublevels of each of the states (with the major contribution from the lower state), as well as the contribution of the MC transfer from the upper state to the lower one, the transfer rate constant being $A_c = -A_{mn}/3$. In this case, the relative contribution of the MC transfer process to the resonance amplitude is determined in accordance with Eqn (11) by the quantity $A_c/(\Gamma_m - \Gamma_n) = -A_{mn}/3\Gamma_m$. Since $A_c < 0$, the MC transfer leads to an increase in absorption at the line centre. The maximum change in absorption occurs in a closed transition (for $A_{mn} = \Gamma_m$) and amounts to $\sim$30% of the resonance amplitude due to the MC levels. In the case of open transitions (for $A_{mn} < \Gamma_m$), the change in the absorption value is much smaller (proportional to the ratio A_{mn}/Γ_m).

For moving gas atoms, the contribution of coherent processes to the work of the probe field in counterpropagating light waves is known to be insignificant (suppressed in the proportion of Γ/kv_T). It manifests itself only at a high saturating fields ($\kappa \gg 1$) [14, 20], whereas in unidirectional waves, this contribution is decisive even at low saturating fields.

In the case of unidirectional waves, when Eqn (10) is averaged over the Maxwell velocity distribution of atoms at large Doppler broadening ($kv_T \gg \Gamma_s$), the work of the probe wave field in the approximation of the first-order nonlinear corrections with respect to the saturating field ($\kappa \ll 1$) for the frequency detunings of the fields $\Omega_\mu \ll kv_T$, $\Omega \ll kv_T$ is determined as

$$P_\mu = 4\hbar\omega_\mu |G^\mu_{+-}|^2 \langle N_{nm}\rangle \frac{\sqrt{\pi}}{kv_T} \exp\left[-\left(\frac{\Omega_\mu}{kv_T}\right)^2\right] \mathrm{Re}(F(\varepsilon)), \quad (12)$$

where

$$F(\varepsilon) = 1 - \frac{\kappa\Gamma}{2\Gamma - \mathrm{i}\varepsilon} - \frac{2|G_+|^2\eta(\varepsilon)}{2\Gamma - \mathrm{i}\varepsilon}; \quad (13)$$

$$\eta = \frac{\Gamma_m - \Gamma_n - A_c}{\Gamma_m - \Gamma_n}\frac{1}{\Gamma_m - \mathrm{i}\varepsilon} + \frac{\Gamma_m - \Gamma_n + A_c}{\Gamma_m - \Gamma_n}\frac{1}{\Gamma_n - \mathrm{i}\varepsilon}. \quad (14)$$

It follows from expressions (12)–(14) that in a weak saturating field, a resonance is formed on the Doppler probe wave absorption contour in the form of a dip with a half-width of 2Γ, centred at $\varepsilon = 0$. Near the line centre, as in the case immobile atoms, structures are formed whose parameters are determined by the values of the relaxation constants of the transition [via the factor $\eta(\varepsilon)$]. When $\Gamma_m \gg \Gamma_n$, we present the factor $\eta(\varepsilon)$ near the line centre (at $\varepsilon/\Gamma_m \ll 1$) in the form

$$\eta(\varepsilon) \approx \frac{1}{\Gamma_m}\left(\frac{\Gamma_m - A_c}{\Gamma_m} + \frac{\Gamma_m - \Gamma_n + A_c}{\Gamma_n - \mathrm{i}\varepsilon}\right). \quad (15)$$

From Eqn (15) it follows that in the broad dip in Eqn (12) near the line centre, a narrow Lorentz structure with half-width Γ_n is formed, the amplitude of which is determined by the level relaxation constants, the first Einstein coefficient A_{mn}, and the upper state MC relaxation constant A_c, like in the case of immobile atoms. For $\Gamma_m \gg \Gamma_n$ the factor $\Gamma_m - \Gamma_n + A_c = \Gamma_m - \Gamma_n - A_{mn}/3$ in Eqn (15) is always positive, and the structure in any transitions manifests itself as a narrow dip.

Let us compare the contributions of the first (population) and second (coherent) terms to the resonance amplitude near the line centre. From Eqns (13) and (14) it follows that the amplitudes of the population and coherent dips are related as $[(1 + 2\Gamma_n/(\Gamma_m + \Gamma_n - A_{mn})]/[(\Gamma_m - \Gamma_n - A_{mn}/3)/(\Gamma_m + \Gamma_n - A_{mn})]$. Hence, in an open transition with $a_0 = 0.5$, the amplitude ratio is approximately 1/2, and in a closed transition (with $a_0 = 1$), the contribution of the MC to the resonance amplitude will significantly exceed the contribution of the incoherent term (in the proportion of $\Gamma_m/3\Gamma_n \gg 1$). The contribution of the transfer of the upper state MC to the lower state in the resonance amplitude in Eqn (12) is determined by the quantity $A_c/(\Gamma_m - \Gamma_n)$. With $A_c = -A_{mn}/3$, the MC transfer contribution leads to an increase in absorption at the line centre (because $A_c < 0$), its maximum value being reached at a closed transition and amounting to $\sim$30% of the amplitude of the narrow resonance structure.

Thus, the nonlinear resonance in a closed transition between the energy levels with angular momenta $J = 1/2$ for orthogonal polarisations of fields and a weak saturating field is exclusively coherent and is due to the MC of the levels (mainly those of the lower state) of the transition. The resonance will also have a coherent nature in a strong saturating field (see below).

2.2. Probe field absorption spectrum in a system of two levels with $J = 1/2$ in the saturating field of arbitrary intensity

To reveal the features of the behaviour of the saturated absorption resonance shape and the processes forming the resonance depending on the intensity of the saturating field and the parameters of the atomic transition, numerical studies of the probe wave absorption spectrum were performed according to expressions (9) and (10) for the exact solution of the problem. In modelling the resonance shape, the contributions from the incoherent effect of saturation of level populations and the coherent effects, such as inducing MC by optical fields and its transfer from the levels of the upper state to the lower one, as well as splitting of the transition levels by the saturating wave field, were determined. The calculations were carried out for the following parameters of the atomic transition: $\Gamma_m = 5.5 \times 10^7$ s^{-1}, $\Gamma_n = (10^{-2} - 1)\Gamma_m$, $\Gamma = (\Gamma_m + \Gamma_n)/2$. The ratio N_m/N_n of the initial level populations was assumed to be $\sim 10^{-2}$. The Doppler linewidth $(kv_T)_D$ was taken to be 5×10^9 s^{-1}. For the integration, the range of particle velocities v was taken $\pm 3(kv_T)_D/k$ with a step $\Delta v = (10^{-3} - 10^{-4}) \times (kv_T)_D/k$, the strong field saturation parameter κ varied within 0.01 – 10, and the branching parameter $a_0 = A_{mn}/\Gamma_m$ varied in the range 0 – 1. The nonlinear resonance shape was determined from the work (10) of the probe field as $\alpha(\Omega_\mu) = \Gamma_{mn}P_\mu \times (4\hbar\omega_\mu |G^\mu_{+-}|^2)^{-1}$.

The calculations showed that solutions of (12), (13), and (14) in the approximation of the first-order nonlinear correction in the strong field intensity practically do not differ from the exact solution for the saturation parameters of the strong field $\kappa < 0.1$. For $\kappa \geqslant 0.2$, the differences between the solutions of both the first-order and the next approximations in the strong field become significant (especially for a closed transition), and it is impossible to use the analytical formulae of these approximations. The calculations also revealed significant differences between the spectra of nonlinear resonances and the resonance-forming processes for open ($a_0 < 1$) and closed ($a_0 = 1$) atomic transitions.

The characteristic shapes of nonlinear resonances, as well as the contributions to the resonance from the level population saturation effect are shown in Fig. 3 for a closed transition at $\Gamma_n/\Gamma_m = 2\times10^{-2}$ and $\kappa = 0.01-5$. It can be seen that the resonance manifests itself on the Doppler profile of the probe wave absorption line as a wide dip and a narrow structure (dip) near the centre of the line, the parameters of which depend on the intensity (saturation parameter κ) of the strong wave. Studies of the resonance-forming processes have shown that the wide dip (dash-dotted lines) is due to the incoherent effect of saturation of the level populations by a strong field, while the narrow structures are due to the contribution of coherent processes. In this case, for any type of transitions, with a change in the saturation parameter in the range of $0.01-5$, the dependences of the amplitude and width of the incoherent dip obey, respectively, linear ($\sim\kappa$) and close to the square root of κ ($\sim\sqrt{1+\kappa}$) laws, as in a system of two levels [14, 20].

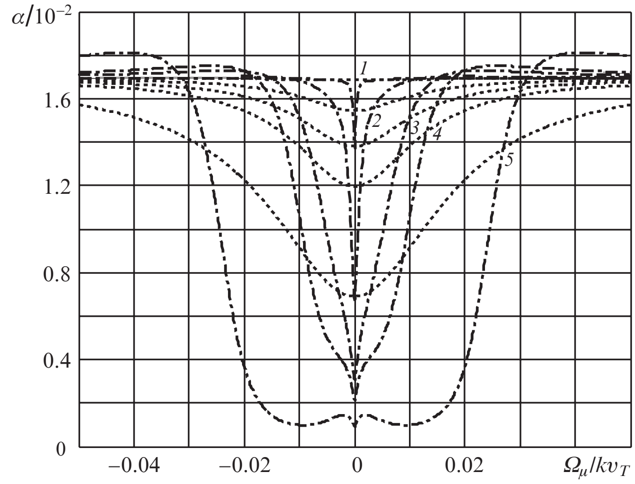

Figure 3. Population parts (dashed line) and total resonances (dash-dotted line) of saturated absorption for a closed ($a_0 = 1$) transition at $\Gamma_n/\Gamma_m = 0.02$, $\kappa = 0.01$ (*1*), 0.1 (*2*), 0.5 (*3*), 1.0 (*4*), 5 (*5*).

In the case of open transitions, coherent processes manifest themselves in a narrow frequency region near the centre of the line and are due to the MC of the levels induced by light fields, mainly in the lower state. Their maximum contribution to the resonance amplitude in this region approximately corresponds to the contribution of the incoherent effect. The change in the intensity of the saturating field manifests itself in changes (within small limits) of the amplitude and width of the narrow structure of the resonance according to a linear law with respect to κ.

In the case of a closed transition (Fig. 3), the resonance spectrum of saturated absorption is formed mainly by the contributions of coherent processes, in which dependences on the intensity (saturation parameter κ) of the strong field characteristic of these processes appear. For example, at low values of this parameter [$\kappa < 0.1$, curves (*1*, *2*)], the contribution of saturation of the level populations to the resonance shape is small, and the resonance spectrum is formed exclusively by coherent processes: the MC of the levels induced by the optical fields and its transfer, which form a dip-shaped narrow structure having the width of the lower level Γ_n.

With an increase in the saturation parameter in the range $\kappa = 0.01 - 0.5$ [curves (*1* – *3*)], the amplitude of the coherent dip and its width increase (according to a law close to linear in κ), and at $\kappa \sim 1$ and more [curve (*4*)] additional structures appear at the line wing. The frequency interval between the maxima of these structures is $\Delta\omega \sim 10^{-2}\times\Delta\omega_D \approx \Gamma_m \approx 2\Gamma$, i.e. it is determined by the width of the transition line adopted in the calculations. With a further increase in κ ($\kappa > 2$), the shape of the resonance has the form of three spectral components [curves (*4, 5*)]. In this case, with an increase in the saturation parameter, the amplitude of the central component decreases, the amplitudes of the side components increase, and the frequency separation between their maxima obeys the root dependence on the parameter κ (linear dependence on the interaction parameter G). This fact indicates that the extreme side components of the spectrum are due to the field splitting of the transition levels.

Figure 4 shows the shapes of the contribution to the resonance from the transfer of the MC from the upper state to the lower state for a closed transition at various values of the strong field saturation parameter. It can be seen that the MC transfer leads to an increase in the absorption coefficient at the centre of the line, and its maximum value with the values of the transition relaxation constants adopted in the calculations is realised at $\kappa \lesssim 0.1$ and amounts to $\sim$30 % of the resonance amplitude. In the case of an open transition with $a_0 = 0.5$, this contribution is much smaller (less than 10% of the narrow structure amplitude). In this case, the behaviour of the shapes of the MC transfer contributions with a change in the saturating field intensity is specific for coherent processes. At $\kappa < 0.05$, the shape of the MC transfer contribution manifests itself as a sign-changing interference structure [curve (*1*)], which is typical for NIEF [14]. At $\kappa = 0.05 - 0.5$, a characteristic field splitting of the spectrum of contributions near the line centre into two components occurs [curves (*2, 3*)], while at $\kappa \geqslant 1$, the spectra acquire a complex shape [curves (*4, 5*)]. Estimation of the splitting in the case of curve (*2*) (the beginning of the effect, $\kappa = 0.05$) yields $\Delta\omega \approx 6\times10^{-2}\Gamma_m \approx 3\Gamma_n$, i.e., it is determined by the half-width of the lower state energy levels. For curve (*4*) ($\kappa = 1$), the splitting is $\Delta\omega \approx 36\Gamma_n \approx 1.4\Gamma$, i.e., it is already determined by the homogeneous half-width of the transition line. This splitting also manifests itself as a total absorption resonance [Fig. 3, curve (*4*)]. Estimates of the interaction parameter G from relation (7) give $G \approx 0.1\Gamma_m \approx 5\Gamma_n$ (at $\kappa = 0.05$) and $G \approx 0.5\Gamma_m \approx \Gamma$ (at $\kappa = 1$).

The analysis of the dependences of the splitting value in the spectra of the MC transfer contribution to the total resonance (Fig. 4) on the saturation parameter in the range $\kappa = 0.01 - 1$ shows that the splitting grows faster than the root dependence on κ, but slower than the linear one. In this case, the minimum splitting in the spectrum of the contribution of the MC transfer, as follows from the estimates, is determined by the relaxation constant of the lower levels Γ_n and is due to the splitting of the levels of the lower state of the transition by the field of the saturating wave. Thus, the spectrum of the MC transfer contribution to the total resonance turns out to be more sensitive to the intensity of the saturating field, since

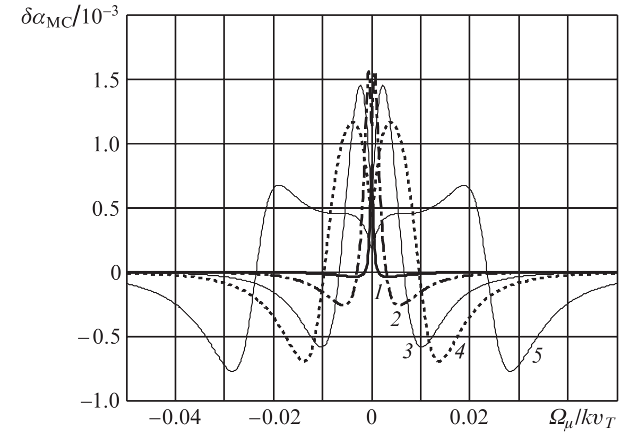

Figure 4. Contributions to the resonance of the effect of MC transfer from the upper state to the lower one for the closed transition ($a_0 = 1$) at $\Omega = 0$, $\Gamma_n/\Gamma_m = 0.02$, and $\kappa = 0.01$ (*1*), 0.1 (*2*), 0.5 (*3*), 1.0 (*4*), 5.0 (*5*).

here the field splitting begins to manifest itself already at the splitting of the levels in the lower long-lived state. The splitting in the total absorption resonance spectrum (see Fig. 3) manifests itself at level splittings larger than the homogeneous transition linewidth, which in the absence of collisions is determined by the relaxation constants of the levels of both the lower and upper states [14].

It should be noted that for closed transitions, in contrast to open ones, the action of the saturating field manifests itself more radically in the shape of resonances both near the centre and in the line wings. Since the incoherent (population) part of the resonance is determined by the saturation parameter κ, and the coherent part is determined by the intensity of the saturating field G^2 [see Eqns (10), as well as (13)and (14)], with the same values of the saturation parameter, the intensity of the saturating field for a closed transition will be $[(1 - a_0)\,\Gamma_m/\Gamma_m + 1]$ times greater than the corresponding intensities for open (with the branching parameter $a_0 < 1$) transitions, which causes the spectral differences between these transitions in the contributions of coherent processes and the shapes of resonances.

2.3. Probe field absorption spectrum in a system of two levels with $J = 1$

A schematic of the interaction of optical fields with magnetic sublevels of the transition between levels with angular momenta $J = 1$ in a coordinate system with the quantisation axis directed along the strong field vector is shown in Fig. 2a. In this system, optical fields form two subsystems with transitions of Λ and V type, interconnected by spontaneous transitions with a change in the magnetic number $M - M' = \pm 1$. In a weak probe field, the population of the upper magnetic sublevel with $M' = 0$ is practically independent of the intensity of the strong field and is equal to the equilibrium value. This circumstance simplifies the analytical consideration of the problem based on solutions for subsystems of Λ and V-type transitions.

Let the subsystems be formed by the magnetic sublevels of the transition p, l, s (upper) and q, k, d (lower). The solutions of Eqns (1) and (2) for off-diagonal elements of the density matrix of a moving atom are sought, according to [14, 21], in the form: $\rho_{pq} = r_{pq}\exp(-\mathrm{i}\Omega t)$, $\rho_{sd} = r_{sd}\exp(-\mathrm{i}\Omega t)$, $\rho_{pk} = r_{pk} \times \exp(-\mathrm{i}\Omega_\mu t)$, $\rho_{kq} = r_{kq}\exp[\mathrm{i}\varepsilon t]$, $\rho_{sk} = r_{sk}\exp(-\mathrm{i}\Omega_\mu t)$, and $\rho_{kd} = r_{kd}\exp[\mathrm{i}\varepsilon t]$ in Λ configurations and $\rho_{lq} = r_{lq} \times \exp(-\mathrm{i}\Omega_\mu t)$, $\rho_{lp} = r_{lp}\exp[-\mathrm{i}\varepsilon t]$, $\rho_{ld} = r_{ld}\exp(-\mathrm{i}\Omega_\mu t)$, and $\rho_{ls} = r_{ls} \times \exp[-\mathrm{i}\varepsilon t]$ in V configurations. Here, the frequency detunings take into account the Doppler shift: $\Omega = \omega - \omega_{mn} - \boldsymbol{kv}$, $\Omega_\mu = \omega_\mu - \omega_{mn} - \boldsymbol{k}_\mu \boldsymbol{v}$, and $\varepsilon = \omega_\mu - \omega - (\boldsymbol{k}_\mu - \boldsymbol{k})\boldsymbol{v}$.

Then in the steady-state case, in the absence of splitting of transition levels, we obtain from Eqns (1), (2) the following systems of algebraic equations for the level populations ρ_i and coherences r_{ij} (for one half of the transition):

$$\begin{aligned}
&\Gamma_m \rho_p = Q_p - 2\,\mathrm{Re}(\mathrm{i}G_1^* r_{pq}),\\
&\Gamma_n \rho_q = Q_q + A_{pq}\rho_p + A_{lq}\rho_l + 2\,\mathrm{Re}(\mathrm{i}G_1^* r_{pq}) + 2\,\mathrm{Re}(\mathrm{i}G_{2\mu}^* r_{lq}),\\
&\Gamma_m \rho_l = Q_l + 2\,\mathrm{Re}(\mathrm{i}G_{2\mu}^* r_{ql}) + 2\,\mathrm{Re}(\mathrm{i}G_{3\mu}^* r_{dl}),\\
&\Gamma_n \rho_k = Q_k + A_{pk}\rho_p + A_{sk}\rho_s + 2\,\mathrm{Re}(\mathrm{i}G_{1\mu}^* r_{pk}) + 2\,\mathrm{Re}(\mathrm{i}G_{4\mu}^* r_{sk}),
\end{aligned} \tag{16}$$

$$\begin{aligned}
&(\Gamma - \mathrm{i}\Omega) r_{pq} = -\mathrm{i}G_1(\rho_p - \rho_q) - \mathrm{i}G_{2\mu} r_{pl} + \mathrm{i}G_{1\mu} r_{kq},\\
&(\Gamma - \mathrm{i}\Omega_\mu) r_{pk} = -\mathrm{i}G_{1\mu}(\rho_p - \rho_k) + \mathrm{i}G_1 r_{qk},\\
&(\Gamma - \mathrm{i}\Omega_\mu) r_{lq} = -\mathrm{i}G_{2\mu}(\rho_l - \rho_q) - \mathrm{i}G_1 r_{lp},\\
&(\Gamma_m - \mathrm{i}\varepsilon) r_{lp} = \mathrm{i}(G_{2\mu} r_{qp} - G_1^* r_{lq}),\\
&(\Gamma_n + \mathrm{i}\varepsilon) r_{kq} = \mathrm{i}(G_{1\mu}^* r_{pq} - G_1 r_{kp}) + A_\mathrm{c} r_{sl}.
\end{aligned} \tag{17}$$

In Eqns (16) and (17), the following notations are introduced for the matrix elements of the interaction operators between the magnetic sublevels of the transition: $G_1 = G_{pq}$, $G_2 = G_{sd}$ for the strong field and $G_{1\mu} = G_{pk}^\mu$, $G_{2\mu} = G_{lq}^\mu$, $G_{3\mu} = G_{ld}^\mu$, and $G_{4\mu} = G_{sk}^\mu$ for the probe field. In the last equation of system (17), the term $A_\mathrm{c} r_{sl}$ describes the spontaneous transfer of the MC from the levels of the upper state to the levels of the lower state. Similar equations are also valid for the second half of the transition.

Eliminating the amplitudes r_{kq} and r_{lp} from Eqns (17), we obtain the equations for the amplitudes of coherences for the $p - q$, $p - k$ and $l - q$ transitions:

$$\begin{aligned}
&\left(\Gamma - \mathrm{i}\Omega + \frac{|G_{1\mu}|^2}{\Gamma_n + \mathrm{i}\varepsilon} + \frac{|G_{2\mu}|^2}{\Gamma_m + \mathrm{i}\varepsilon}\right) r_{pq} = -\mathrm{i}G_1(\rho_p - \rho_q)\\
&\qquad + \frac{G_1 G_{2\mu} r_{lq}^*}{\Gamma_m + \mathrm{i}\varepsilon} + \frac{G_1 G_{1\mu} r_{kp}}{\Gamma_n + \mathrm{i}\varepsilon} + \frac{A_\mathrm{c} G_{3\mu}(G_{3\mu}^* r_{sd} - G_2 r_{dl})}{(\Gamma_n + \mathrm{i}\varepsilon)(\Gamma_m + \mathrm{i}\varepsilon)},\\
&\left(\Gamma - \mathrm{i}\Omega_\mu + \frac{|G_1|^2}{\Gamma_n - \mathrm{i}\varepsilon}\right) r_{pk} = -\mathrm{i}G_{1\mu}(\rho_p - \rho_k) + \frac{G_1 G_{1\mu} r_{pq}^*}{\Gamma_n - \mathrm{i}\varepsilon}\\
&\qquad - \frac{A_\mathrm{c} G_1(G_{3\mu} r_{sd}^* - G_2^* r_{ld})}{(\Gamma_n - \mathrm{i}\varepsilon)(\Gamma_m - \mathrm{i}\varepsilon)},\\
&\left(\Gamma - \mathrm{i}\Omega_\mu + \frac{|G_1|^2}{\Gamma_m - \mathrm{i}\varepsilon}\right) r_{lq} = -\mathrm{i}G_{2\mu}(\rho_l - \rho_q) + \frac{G_1 G_{2\mu} r_{pq}^*}{\Gamma_m - \mathrm{i}\varepsilon}.
\end{aligned} \tag{18}$$

Similar equations are also valid for the amplitudes of coherences in the second half of the transition with the appropriate change of the indices $p \leftrightarrow s$, $q \leftrightarrow d$, as well as the indices of the matrix elements of the interaction operators $G_1 \leftrightarrow G_2$, $G_{1\mu} \leftrightarrow G_{4\mu}$, and $G_{2\mu} \leftrightarrow G_{3\mu}$.

For the transition considered, the rates of spontaneous population and MC transfer between magnetic sublevels are as follows: $A_{lk} = 0$, and $A_{pq} = A_{pk} = A_{sd} = A_{sk} = A_{lq} = A_{ld} = A_c = A_{mn}/2$. Under uniform pumping of the sublevels of the upper and lower states ($Q_p = Q_s = Q_l$ and $Q_q = Q_d = Q_k$) in the absence of external radiation fields, there are identical population differences between them: $\Delta N_{qp} = \Delta N_{ql} = \Delta N_{kp} = \Delta N_{ks} = \Delta N_{dl} = \Delta N_{ds} = \Delta N$.

Solutions (16)–(18) for arbitrary fields are difficult to analyse, therefore, as for the transition between levels with angular momenta $J = 1/2$, we will further consider the case of a weak probe field ($G_\mu \ll G$). In this case, the energy level populations in Eqns (16) are determined by the strong field only, and Eqns (17) and (18) can be presented in a simpler form

$$(\Gamma - \mathrm{i}\Omega_1) r^{\,0}_{pq} = -\mathrm{i}G_1(\rho^0_p - \rho^0_q),$$

$$\left(\Gamma - \mathrm{i}\Omega_\mu + \frac{|G_1|^2}{\Gamma_n - \mathrm{i}\varepsilon}\right) r_{pk} = -\mathrm{i}G_{1\mu}\left[(\rho^0_p - \rho^0_k) - \frac{|G_1|^2[(\rho^0_p - \rho^0_q)}{(\Gamma_n - \mathrm{i}\varepsilon)(\Gamma + \mathrm{i}\Omega)}\right] - \frac{A_c G_1\left(G_{3\mu} r^{\,0}_{ds} - G^*_2 r_{ld}\right)}{(\Gamma_n - \mathrm{i}\varepsilon)(\Gamma_m - \mathrm{i}\varepsilon)}, \qquad (19)$$

$$\left(\Gamma - \mathrm{i}\Omega_\mu + \frac{|G_1|^2}{\Gamma_m - \mathrm{i}\varepsilon}\right) r_{lq} = -\mathrm{i}G_{2\mu}\left[(\rho^0_l - \rho^0_q) - \frac{|G_1|^2[(\rho^0_p - \rho^0_q)}{(\Gamma_m - \mathrm{i}\varepsilon)(\Gamma + \mathrm{i}\Omega)}\right].$$

Here ρ^0_i are solutions of the system of equations that follows from (16) in the case when only the strong field acts; these solutions were derived by us in Ref. [22]. From them it follows that the level population differences are determined by the relations:

$$\rho^0_q - \rho^0_p = \rho^0_d - \rho^0_s = \Delta N\left(1 + \frac{\Gamma_m - \Gamma_n - A_{mn}/2}{\Gamma_m + \Gamma_n - A_{mn}/2}\,\frac{\kappa_1 \Gamma^2}{\Gamma^2_{s1} + \Omega^2}\right),$$

$$\rho^0_k - \rho^0_p = \rho^0_k - \rho^0_s = \Delta N\left(1 + \frac{A_{mn} - \Gamma_n}{\Gamma_m + \Gamma_n - A_{mn}/2}\,\frac{\kappa_1 \Gamma^2}{\Gamma^2_{s1} + \Omega^2}\right), \qquad (20)$$

$$\rho^0_q - \rho^0_l = \rho^0_d - \rho^0_l = \Delta N\left(1 + \frac{\Gamma_m - A_{mn}/2}{\Gamma_m + \Gamma_n - A_{mn}/2}\,\frac{\kappa_1 \Gamma^2}{\Gamma^2_{s1} + \Omega^2}\right),$$

where $\Gamma_{s1} \equiv \Gamma\sqrt{1 + \kappa_1}$ is the saturated transition width; $\kappa_1 = \kappa(1 + A_{mn}/2\gamma_{mn})$; $\gamma_{mn} = \Gamma_m + \Gamma_n - A_{mn}$; $\kappa = 2|G_1|^2\gamma_{mn}/\Gamma\Gamma_m\Gamma_n$ is the transition saturation parameter; and ΔN is the population difference between the sublevels of the lower and upper states, similar for all sublevels under the uniform excitation of the energy levels.

The probe field absorption spectrum, according to [14], is determined by its work

$$P_\mu = -2\hbar\omega_\mu \mathrm{Re}\left\langle \mathrm{i}\sum_{i,k} G^*_{\mu ik} r_{ki}\right\rangle.$$

Then, using the solutions of the system of equations (19), we can express the work of the probe field for one half of the transition in the form $P_\mu = P^0_\mu + \delta P_\mu$, where P^0_μ is the work of the probe field in the absence of the MC transfer, and δP_μ is the addition to the work of the probe field due to the transfer of the MC:

$$P^0_\mu = 2\hbar\omega_\mu \times \mathrm{Re}\left\langle |G_{1\mu}|^2 \frac{(\Gamma_n - \mathrm{i}\varepsilon)}{\Delta_1}\left[(\rho^0_k - \rho^0_p) - \frac{|G_1|^2(\rho^0_q - \rho^0_p)}{(\Gamma_n - \mathrm{i}\varepsilon)(\Gamma + \mathrm{i}\Omega)}\right] + |G_{2\mu}|^2 \frac{(\Gamma_m - \mathrm{i}\varepsilon)}{\Delta_2}\left[(\rho^0_q - \rho^0_l) - \frac{|G_1|^2(\rho^0_q - \rho^0_p)}{(\Gamma_m - \mathrm{i}\varepsilon)(\Gamma + \mathrm{i}\Omega)}\right]\right\rangle, \qquad (21)$$

$$\delta P_\mu = 2\hbar\omega_\mu A_c \times \mathrm{Re}\left\langle \frac{G^*_{1\mu}G_{3\mu}G_1G^*_2}{\Delta_1\Delta_3}\left[(\rho^0_d - \rho^0_l) + \frac{(\rho^0_d - \rho^0_s)(\Gamma - \mathrm{i}\Omega_\mu)}{(\Gamma + \mathrm{i}\Omega)}\right]\right\rangle, \qquad (22)$$

where $\Delta_1 = (\Gamma_n - \mathrm{i}\varepsilon)(\Gamma - \mathrm{i}\Omega_\mu) + |G_1|^2$; $\Delta_2 = (\Gamma_m - \mathrm{i}\varepsilon)(\Gamma - \mathrm{i}\Omega_\mu) + |G_1|^2$; $\Delta_3 = (\Gamma_m - \mathrm{i}\varepsilon)(\Gamma - \mathrm{i}\Omega_\mu) + |G_2|^2$; and the brackets $\langle\ldots\rangle$ denote averaging over the Maxwell velocity distribution of atoms.

The contributions of the second half of the transition are determined by the same expressions with the appropriate change of indices of the matrix elements of the interaction operators and the density matrix, as well as the replacement $\Delta_2 \leftrightarrow \Delta_3$ and $\Delta_1 \to \Delta_4 = (\Gamma_n - \mathrm{i}\varepsilon)(\Gamma - \mathrm{i}\Omega_\mu) + |G_2|^2$. Because of the problem symmetry, the contribution of the second half of the transition yields a factor of 2 in the expression of the total work of the probe field.

Further, we consider the main regularities of the MC effect manifestation in the probe wave absorption spectrum within the approximation of first-order corrections with respect to the saturating field. In this approximation, when expressions (21) and (22) are averaged over the velocities of atoms upon a large Doppler broadening, $\Gamma\sqrt{1 + \kappa_1} \ll kv_T$, and field frequency detunings $\Omega_\mu \ll kv_T$, $\Omega \ll kv_T$ the expression for the work of the probe fields can be represented in the form

$$P_\mu = 8\hbar\omega_\mu |G_{1\mu}|^2 \frac{\sqrt{\pi}}{kv_T}\Delta N\{1 + |G_1|^2 \mathrm{Re}[F_0(\varepsilon)]\} + \delta P_\mu, \qquad (23)$$

where

$$F_0(\varepsilon) = \left[C_0 + \left(\frac{1}{2\Gamma - \Gamma_n} + \frac{1}{2\Gamma - \Gamma_m}\right)\right]\frac{1}{2\Gamma - \mathrm{i}\varepsilon} - \frac{1}{2\Gamma - \Gamma_m}\,\frac{1}{\Gamma_m - \mathrm{i}\varepsilon} - \frac{1}{2\Gamma - \Gamma_n}\,\frac{1}{\Gamma_n - \mathrm{i}\varepsilon} \qquad (24)$$

is the function that determines the structure of the spectrum in the absence of the MC transfer; and

$$C_0 = (\Gamma_m - \Gamma_n + A_{mn}/2)/(\Gamma_m\Gamma_n);$$

$$\delta P_\mu = 8\hbar\omega_\mu A_c \Delta N \frac{\sqrt{\pi}}{kv_T}\mathrm{Re}\left[\frac{G^*_{1\mu}G_{3\mu}G_1G^*_2}{(2\Gamma - \mathrm{i}\varepsilon)(\Gamma_n - \mathrm{i}\varepsilon)(\Gamma_m - \mathrm{i}\varepsilon)}\right] \qquad (25)$$

is the contribution of the MC transfer process. Note that in expressions (22) and (25) the contribution of the MC transfer is determined by the product of the matrix elements of the operator of interaction of the atom with fields $G^*_{1\mu}G_{3\mu}G_1G^*_2$ or by the product $G^*_{4\mu}G_{2\mu}G_2G^*_1$ (for the second half of the transition). Such dependence of the operator of interaction between the fields and the transition on the matrix elements is characteristic of interference processes. For products of matrix elements of the transition between energy levels with angular momenta $J = 1$, according to [3], the equality $G_{1\mu}^{*}G_{3\mu}G_1G_2^{*} = G_{4\mu}^{*}G_{2\mu}G_2G_1^{*} = |G_{1\mu}|^2|G_1|^2$ holds.

It follows from relations (23) and (24) that in the absence of the MC transfer, the absorption spectrum of the probe field for the considered transition consists of a Doppler contour and three resonance structures centred near the field frequency difference $\varepsilon = 0$ and having different amplitudes and widths. The first resonance structure [the first term in Eqn (24)] is represented by a Lorentzian with a half-width of 2Γ and an amplitude determined by the relaxation constants of the transition and the intensity of the strong wave. Since the signs of the amplitude terms are positive, this structure manifests itself as an absorption peak. The peak maximum is realised at a closed transition (at $\Gamma_m = A_{mn}$). Note that at $\Gamma_m \gg \Gamma_n$ the main contribution to the formation of this peak structure is made by the V transition scheme.

The other two structures in Eqn (24) are represented by Lorentzians with half-widths of the upper (Γ_m) and lower (Γ_n) levels and form two dips with equal amplitudes in the work of the probe field.

In the case of radiative level relaxation at $\Gamma_m \gg \Gamma_n$, $2\Gamma = \Gamma_m + \Gamma_n \approx \Gamma_m$, the function $F_0(\varepsilon)$ takes the form

$$F_0(\varepsilon) \approx \frac{\Gamma_m + A_{mn}/2}{\Gamma_m\Gamma_n}\frac{1}{2\Gamma - i\varepsilon} - \frac{1}{2\Gamma - \Gamma_n}\frac{1}{\Gamma_n - i\varepsilon}. \quad (26)$$

In this case, the absorption spectrum will consist of a broad (with a half-width of 2Γ) absorption peak, which is formed as a result of the addition (subtraction) of two broad Lorentz-type contours with different signs of the amplitudes and a narrow dip with a half-width Γ_n. The ratio of the amplitudes of the peak structure and the dip is determined as $\sim 1 + A_{mn}/2\Gamma_m$. The maximum of this ratio (1.5) is realised for a closed transition (for $A_{mn} = \Gamma_m$).

The contribution of the MC transfer (25) to the work of the probe field, considering the relations for matrix elements, takes the form:

$$\delta P_\mu = 8\hbar\omega_\mu \Delta N \frac{\sqrt{\pi}}{kv_T}|G_{1\mu}|^2|G_1|^2\frac{A_c}{\Gamma_m - \Gamma_n} \times \mathrm{Re}\left[\frac{1}{(2\Gamma - i\varepsilon)}\left(\frac{1}{\Gamma_n - i\varepsilon} - \frac{1}{\Gamma_m - i\varepsilon}\right)\right]. \quad (27)$$

Then expression (23) for the operation of the probe field reads as follows:

$$P_\mu = 8\hbar\omega_\mu|G_{1\mu}|^2\frac{\sqrt{\pi}}{kv_T}\Delta N\{1 + |G_1|^2\mathrm{Re}[F(\varepsilon)]\}, \quad (28)$$

where

$$F(\varepsilon) = \left[C_0 + \frac{1}{2\Gamma - \Gamma_n}\left(1 - \frac{A_c}{\Gamma_m - \Gamma_n}\right) + \frac{1}{2\Gamma - \Gamma_m}\left(1 + \frac{A_c}{\Gamma_m - \Gamma_n}\right)\right] \times \frac{1}{2\Gamma - i\varepsilon} - \frac{1}{2\Gamma - \Gamma_m}\left(1 + \frac{A_c}{\Gamma_m - \Gamma_n}\right)\frac{1}{\Gamma_m - i\varepsilon} - \frac{1}{2\Gamma - \Gamma_n}\left(1 - \frac{A_c}{\Gamma_m - \Gamma_n}\right)\frac{1}{\Gamma_n - i\varepsilon}. \quad (29)$$

It follows from expression (29) that the MC transfer manifests itself both in the amplitude of the population part of the resonance and in the amplitudes of its structures: the amplitudes of wide structures increase, while the amplitude of the narrow dip decreases.

In the case of radiative relaxation of levels and $\Gamma_m \gg \Gamma_n$, the function $F(\varepsilon)$ has the form:

$$F(\varepsilon) \approx \left(\frac{\Gamma_m + A_{mn}/2}{\Gamma_m\Gamma_n} - \frac{A_c}{\Gamma_m^2}\right)\frac{1}{2\Gamma - i\varepsilon} - \frac{1}{\Gamma_m}\left(1 - \frac{A_c}{\Gamma_m}\right)\frac{1}{\Gamma_n - i\varepsilon}. \quad (30)$$

As follows from Eqn (30), the MC transfer effect leads to a decrease in the amplitude of the wide peak and the amplitude of the narrow resonance dip. Moreover, for the peak structure, the relative change in the amplitude is small, $\sim A_c\Gamma_n/\Gamma_m^2$, while for a narrow dip it is $\sim A_c/\Gamma_m$ and can reach $\sim 50\%$ in the case of a closed transition (at $A_{mn} = \Gamma_m$, and $A_c = A_{mn}/2$).

Thus, the MC effect with $M - M' = \pm 1$ (the so-called optical orientation, see Fig. 2a) at the transition between energy levels with angular momenta $J = 1$ forms in the probe wave absorption spectrum a narrow structure (dip) with a half-width, equal to the half-width of the lower level. The MC transfer only reduces the amplitude of the resonance without qualitatively affecting its shape. Note that the MC with $M - M' = \pm 2$ (the so-called optical alignment, Fig. 2b) at this transition, according to our paper [16], manifests itself in a complex way. In the case of parallel polarisations of the radiation fields, the MC of levels forms a wide dip in the resonance spectrum with a width equal to the width of the transition line. In the case of orthogonal polarisations, it forms a narrow dip with a width equal to the width of the lower level. The contribution of the upper level MC transfer for parallel polarisations leads to a decrease in absorption, and for orthogonal polarisations – to an increase in absorption at the line centre. For any wave polarisations, the maximum changes in the resonance amplitude because of the MC transfer do not exceed 10%; the MC transfer also does not affect the qualitative form of the nonlinear resonance.

3. Conclusions

The presented analytical and numerical studies of the saturated absorption spectra of transitions between levels with angular momenta $J = 1/2$ and $J = 1$ in the probe field method with unidirectional orthogonally polarised laser waves demonstrate their dependence on the relaxation constants Γ_m and Γ_n of the levels, the degree of openness a_0 of the atomic transition, and the strong wave intensity.

It is shown that the specific features of the resonance spectra are formed in Λ configurations and manifest themselves as narrow structures – dips of a coherent nature, against the background of contributions from incoherent processes. The physics of incoherent processes and the form of their contributions to the resonance of saturated absorption for these transitions are different. For the transition between the energy levels with the angular momenta $J = 1/2$, this is the effect of saturation of the level populations by the field of the strong wave, which forms a wide dip in the Doppler probe field absorption contour. For the transition between the levels with $J = 1$, this is the effect of optical pumping, giving rise to nonequilibrium population of the lower state magnetic sublevels and, as a consequence, to a broad absorption peak in the form of the resonance. The dip and peak parameters are determined by the constants of relaxation of the levels Γ_m, Γ_n and the coherence Γ, the intensity of the saturating field, and the branching parameter a_0.

The reason for the appearance of narrow (with the width Γ_n of the lower level) structures (dips) of resonances is the MC of the levels (the so-called optical orientation) induced by optical fields and its transfer from the upper state to the lower one. The main contribution to the resonance amplitude is made by the levels of the lower state; the contribution of the MC transfer from the levels of the upper state to the lower state manifests itself only as an addition to the absorption near the centre of the line. The maximum change in absorption occurs at a closed transition (at $A_{mn} = \Gamma_m$) and can be $\sim$30% of the resonance amplitude at the transition between levels with angular momenta $J = 1/2$ and $\sim$50% at the transition between levels with angular momenta $J = 1$. In the case of open transitions (for $A_{mn} < \Gamma_m$), the absorption changes significantly less (in proportion to the A_{mn}/Γ_m ratio). The amplitudes of the population part of the resonance and coherent structures at the transition between levels with angular momenta $J = 1$ are close in magnitude, and in the case of a transition between levels with angular momenta $J = 1/2$, the contribution of coherent processes significantly exceeds the population part of the resonance, and the resonance is exclusively coherent.

The form of the contribution of the MC transfer to the total resonance has an interference character typical for coherent processes. The spectrum of the MC transfer contribution, as in the case of a transition between levels with angular momenta $J = 1/2$, turns out to be more sensitive to the intensity of the saturating field, since the strong field influence on the shape of the transfer contribution begins to manifest itself already at intensities that cause splitting of the levels of the lower long-lived state, whereas in the total resonance spectrum this effect manifests itself in resonance wings at level splittings exceeding the homogeneous width of the transition line. It is important to note that the effect of the transfer of the MC levels (optical orientation) for the considered transitions, as well as the effect of optical alignment studied in [16], does not qualitatively affect the shape of the nonlinear resonance and manifests itself only quantitatively.

In conclusion, we note that the above results were obtained for monochromatic light fields. It was shown in Ref. [23] that when optical coherence is induced between magnetic sublevels in a resonant medium with a transition of the Λ type by saturating light beams with a finite spectral width due to phase fluctuations, correlations of intensity fluctuations of these beams arise. In this case, the width of the intensity correlation resonance can be 25% of the EIT resonance width in the medium. Therefore, the problem of the influence of the finite width of the spectrum of optical fields on the limiting parameters of narrow structures of MC resonances at transitions between levels with angular momenta $J = 1/2$ and $J = 1$ is important and will be considered in future.

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